ACCQ206 exercises – Week 5

Peter Brown

March 29, 2023

1. Recall the bitflip code was a generalisation of the repetition code designed to correct X errors that occur on a single qubit. Suppose an error occurs on the first qubit, can the bitflip code correct the error if the error is:

   (a) A Pauli Y error? Recall $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

   (b) An error of the form
   $$U_\theta = \begin{pmatrix} \cos(\theta) & i\sin(\theta) \\ i\sin(\theta) & \cos(\theta) \end{pmatrix} \tag{1}$$
   for any $\theta \in [0, 2\pi)$.

2. Design a quantum circuit that implements the error correction step for the 3 qubit bit-flip code but does NOT use measurements. (Hint: you can create gates with multiple target qubits – e.g., CNOTS with more than one target (Toffoli gates))

3. Draw the circuit that implements the encoding for Shor's 9 qubit code.

4. Show how to do the error detection/correction step in the Shor 9 qubit code when the first qubit undergoes both a phase-flip error and a bit-flip error.

5. The Shor code is a $[[9, 1, 3]]$ code, i.e., it has distance 3. This means that there exists 2 different valid codewords $|\psi_0\rangle$, $|\psi_1\rangle$ and an error E that acts on 3 qubits such that $|\psi_0\rangle = E\,|\psi_1\rangle$. Find an explicit example.

6. (More challenging)

   Consider the $[[5, 1, 3]]$ stabilizer code we saw in class. It was constructed from the stabilizer subgroup generated by
   $$\begin{aligned} S_1 &= X_1 Z_2 Z_3 X_4 \\ S_2 &= X_2 Z_3 Z_4 X_5 \\ S_3 &= X_1 X_3 Z_4 Z_5 \\ S_4 &= Z_1 X_2 X_4 Z_5 \end{aligned} \tag{2}$$
   Construct a valid encoding map $|0\rangle \mapsto |\psi_0\rangle$ and $|1\rangle \mapsto |\psi_1\rangle$ for this code, i.e., find such a $|\psi_0\rangle$ and $|\psi_1\rangle$ that are an orthogonal basis for the stabilizer subspace.