Variational bounds on the relative entropy and their applications

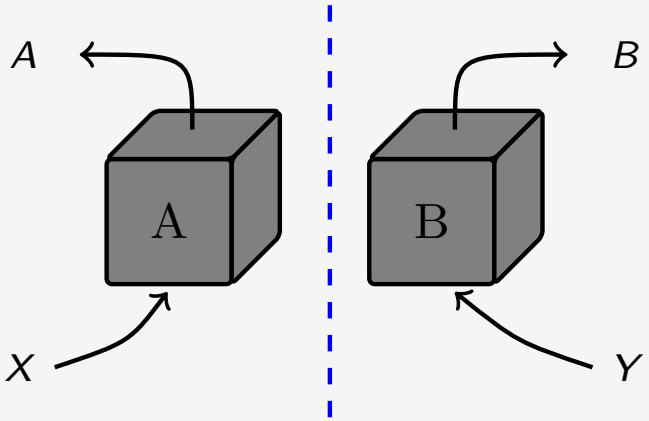
Peter Brown, Hamza Fawzi and Omar Fawzi

Paper 1: arXiv:2106.13692
Paper 2: arXiv this week

Mar 07 2022

Motivation – Device-independence

Bell-nonlocality



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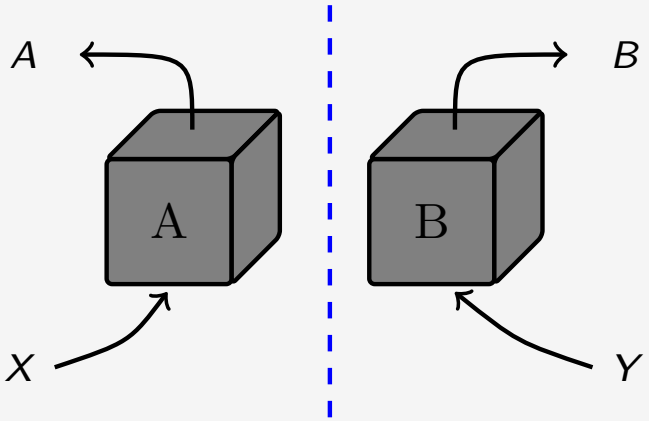

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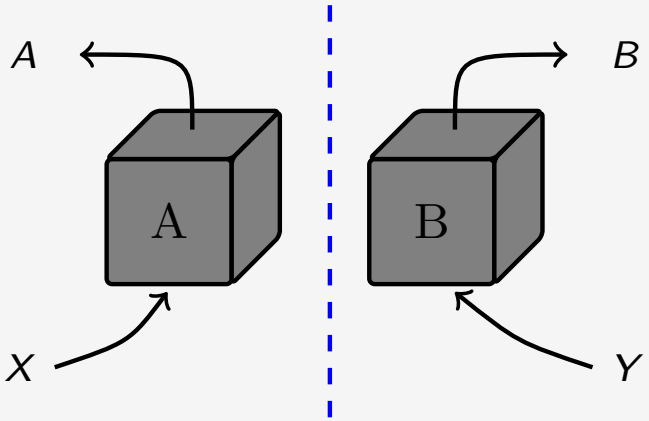

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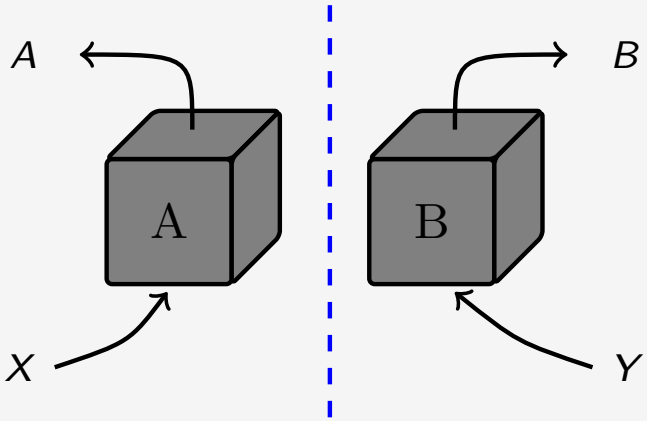

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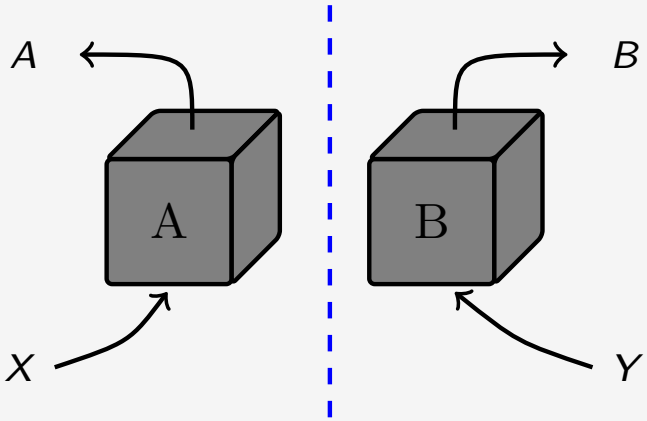

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Optimization of von Neumann entropies

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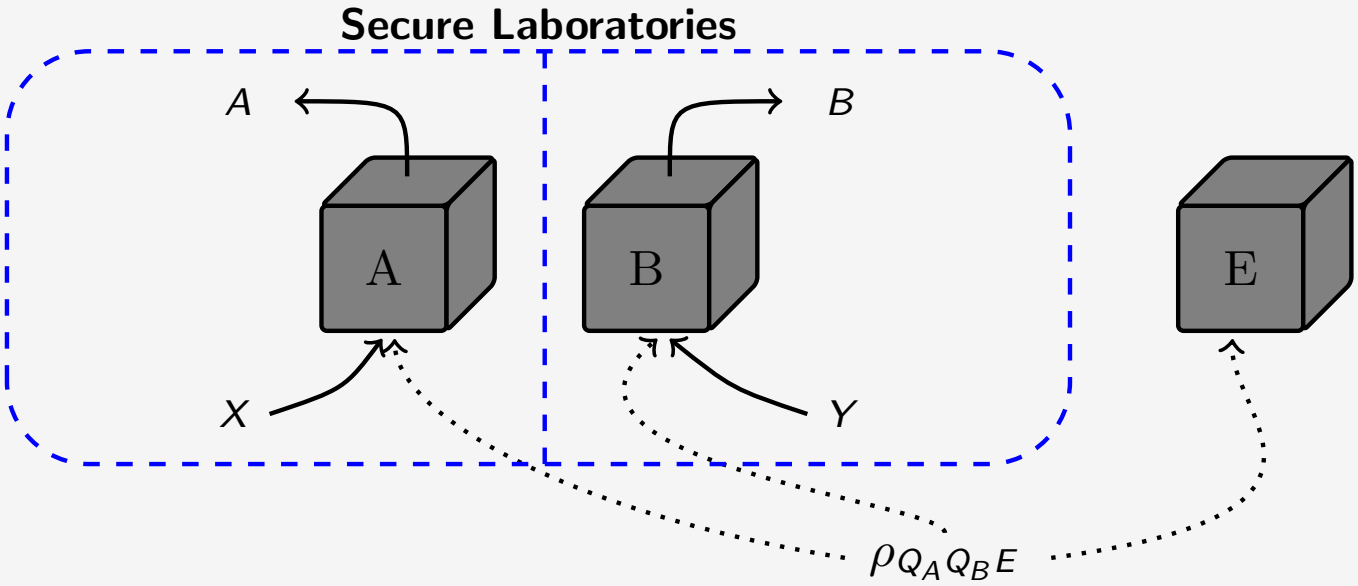

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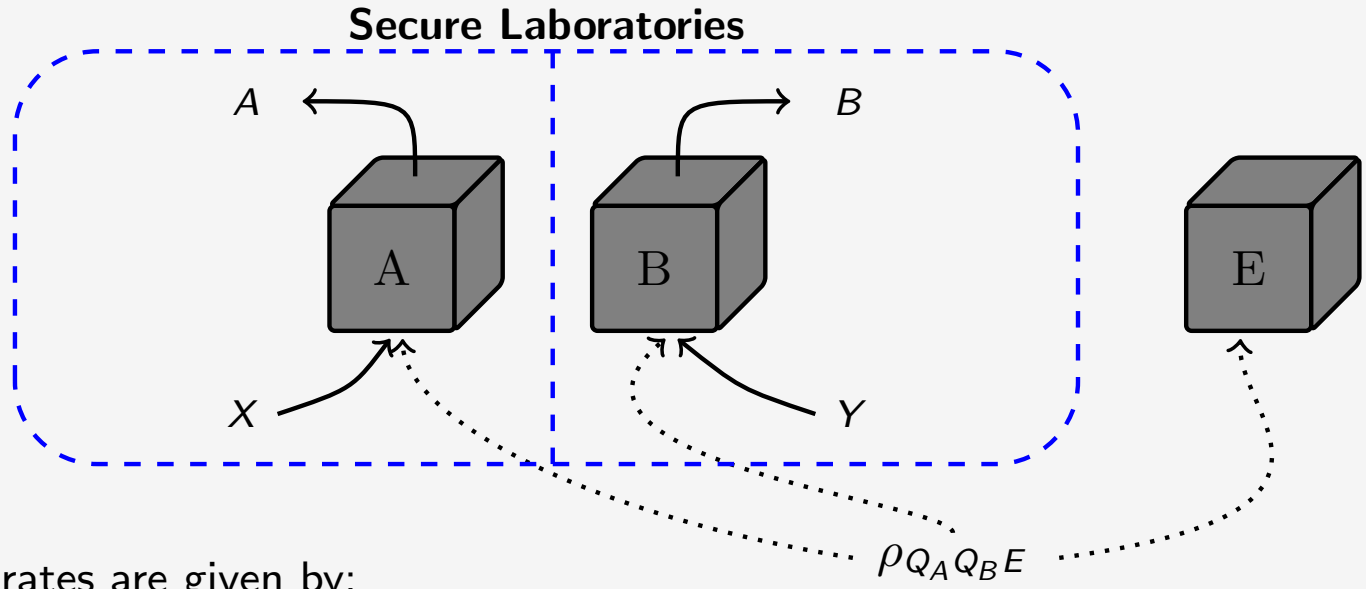

Asymptotic rates are given by:

- **Randomness expansion**

$$H(AB|X = x^*, Y = y^*, E)$$

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$$H(A|X = x^*, E) - H(A|X = x^*, Y = y^*, B)$$

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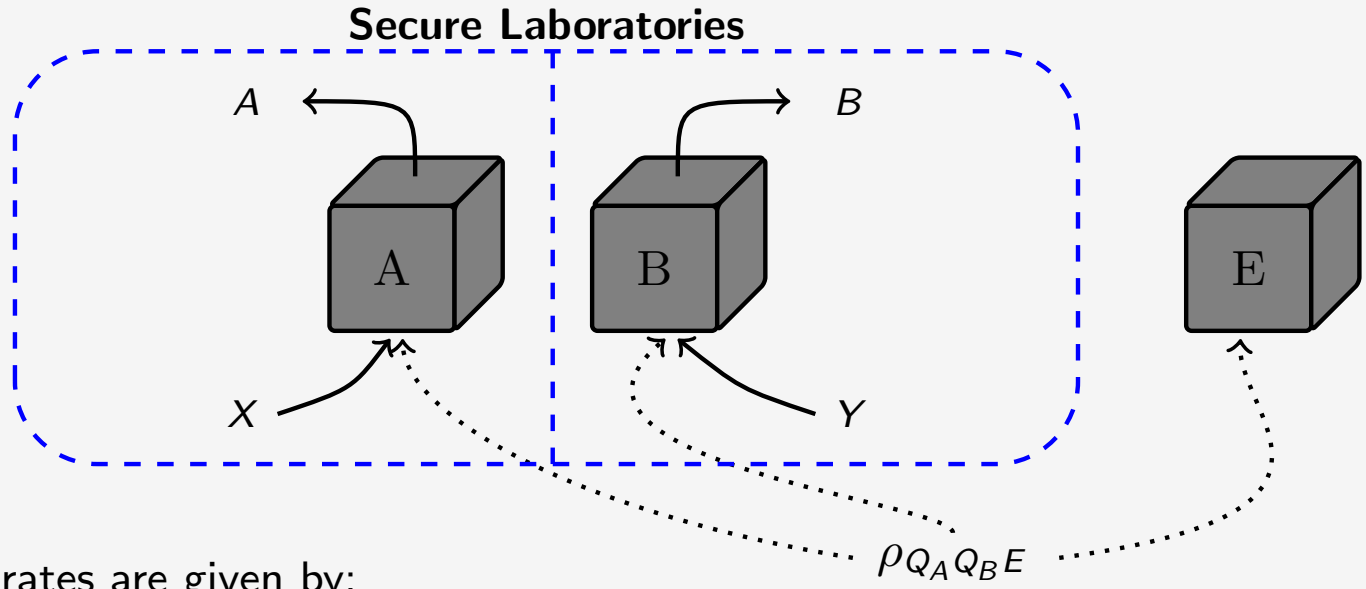

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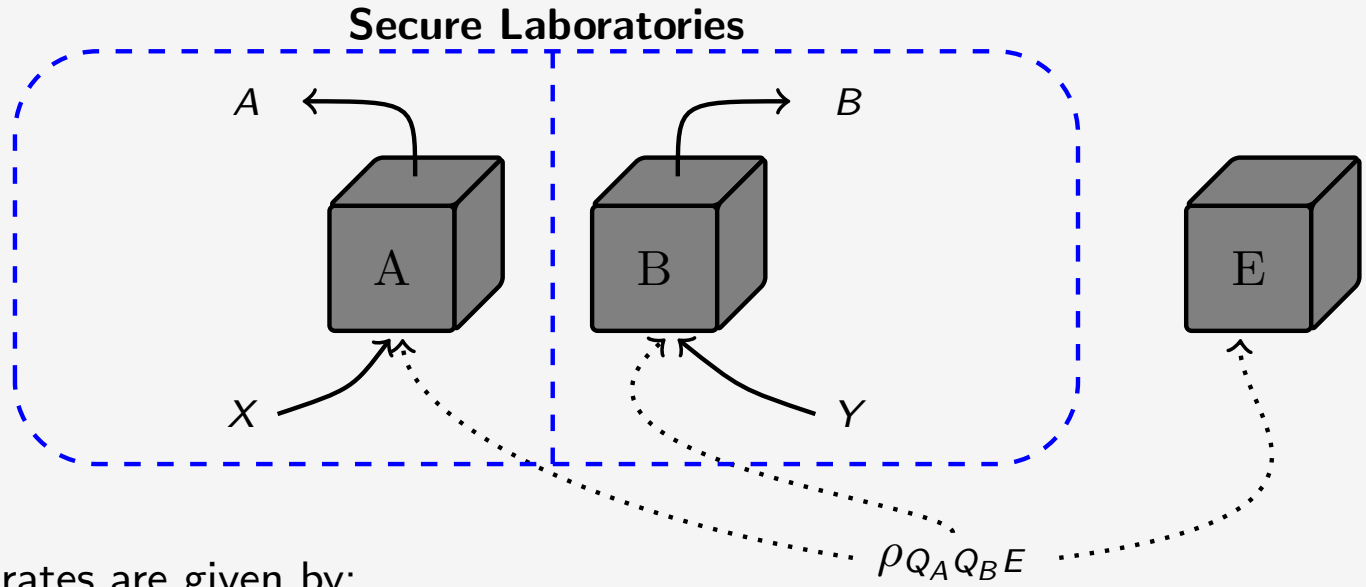

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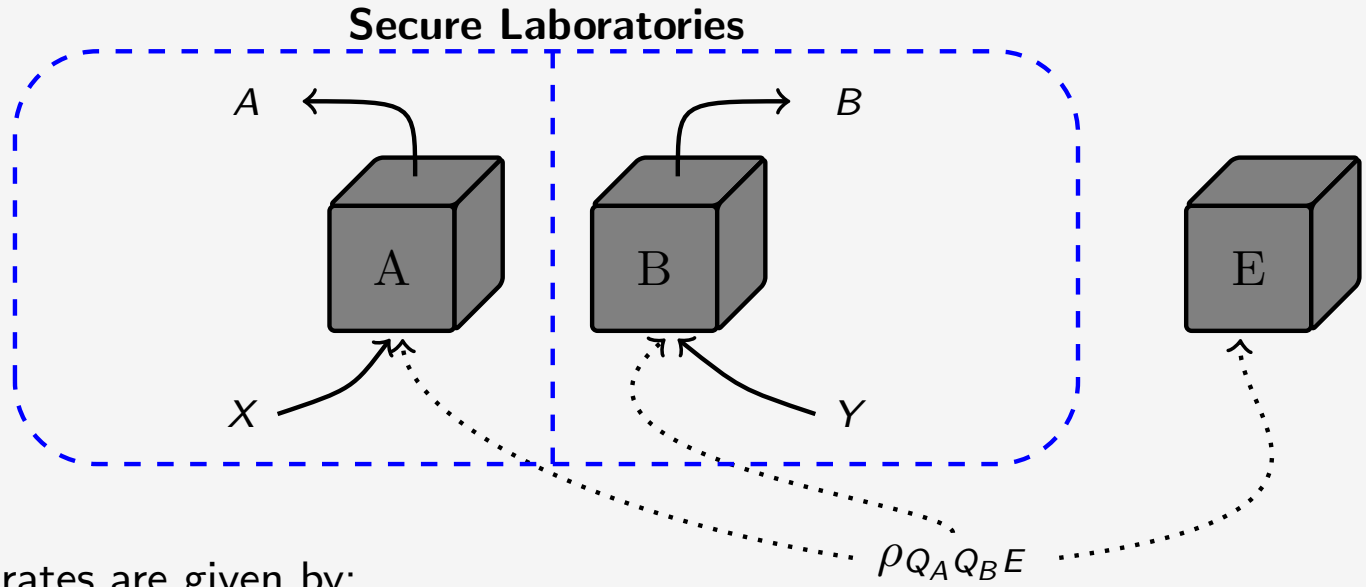

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Noncommutative polynomial optimization problems

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infimum over $(\mathcal{H}, \rho, Z_1, \ldots, Z_n)$.

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Goal:

Search for variational bounds on entropies with an NCPOP form.

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The goal

Derive something of the form

$$D(\rho\|\sigma) \leq \sum_{i=1}^{m} \sup_{Z} \mathrm{Tr}\left[\rho p_i(Z)\right] + \mathrm{Tr}\left[\sigma q_i(Z)\right]$$

with p_i and q_i some polynomials and with the RHS converging as $m \to \infty$.

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Know $D(\rho\|\sigma) = \sup_{(X,Y,z)\in\mathcal{F}} \mathrm{Tr}\left[\rho X\right] + \mathrm{Tr}\left[\sigma Y\right] + z$

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Derivation overview

1. Gauss-Radau approximation of the logarithm

$$\ln(x) = \int_0^1 \frac{x-1}{t(x-1)+1} \mathrm{d}t \geq \sum_{i=1}^{m} w_i f_{t_i}(x)$$

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Main Result

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and RHS converges as $m \to \infty$.

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$$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

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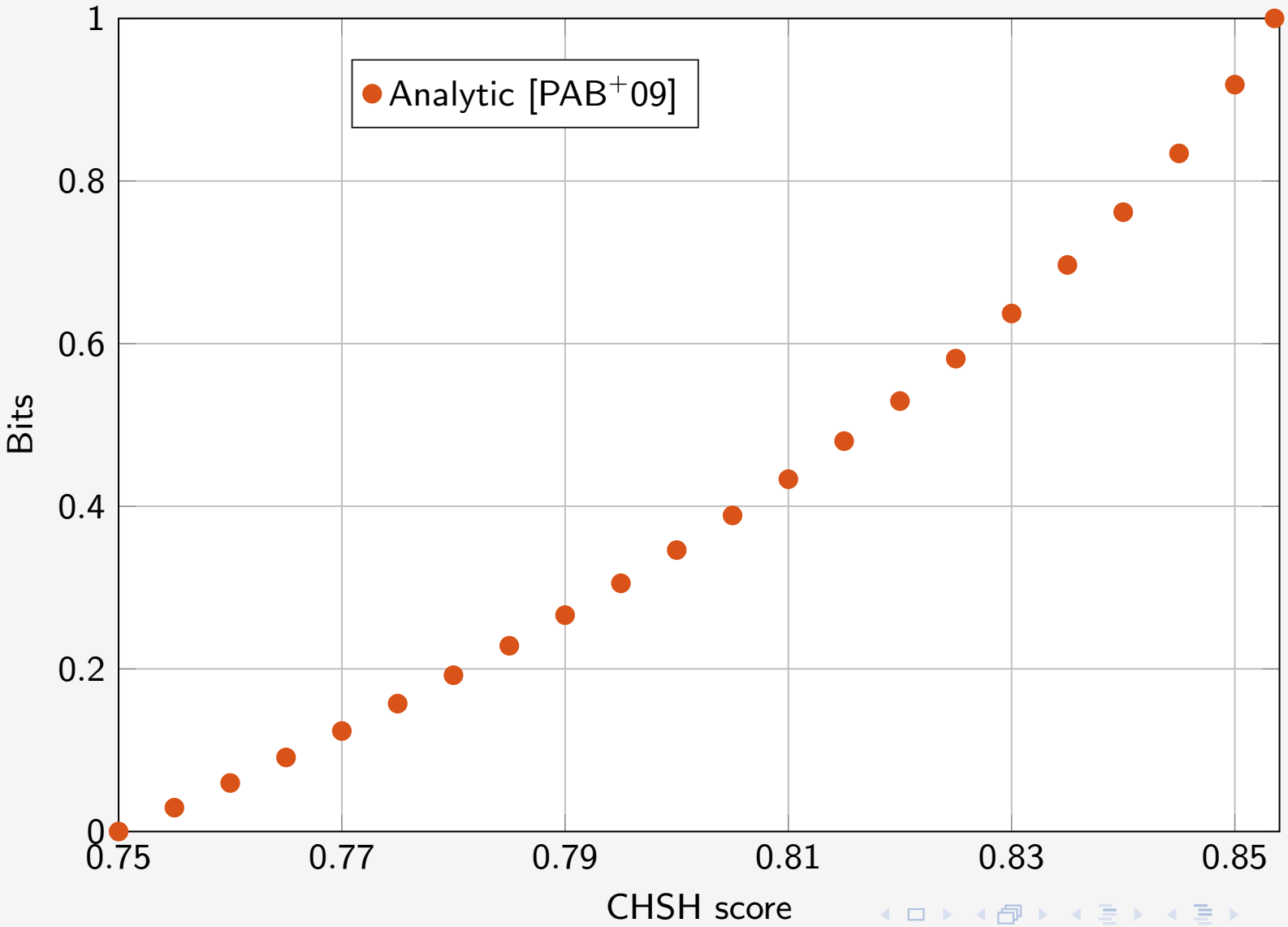
- Similar results for $H(AB|X = x, Y = y, Q_E)$ or $H(A|XQ_E)$ and others.

Results I – Recovering tight bounds for the CHSH game

Bounding $\inf H(A|X=0, Q_E)$



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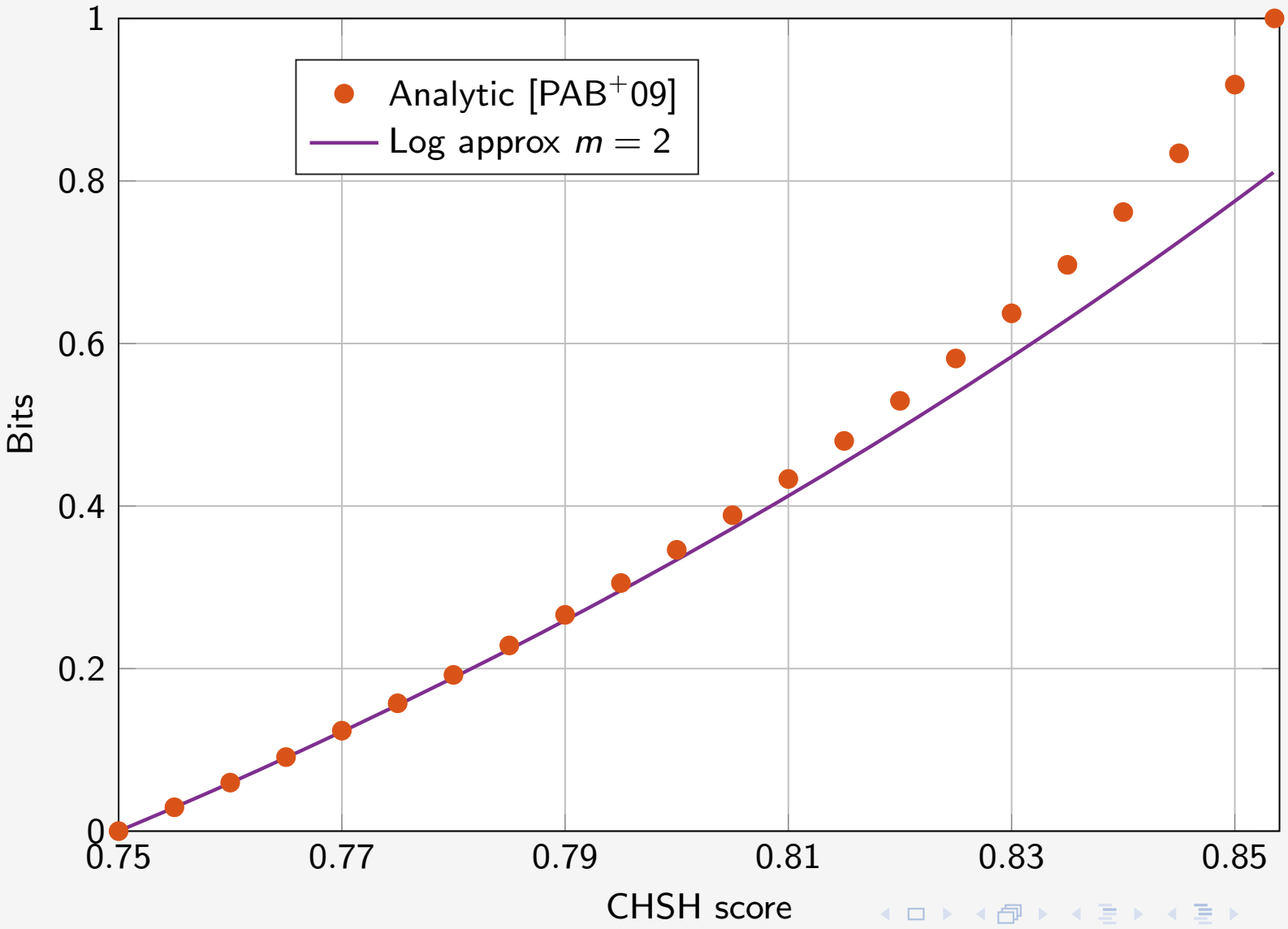

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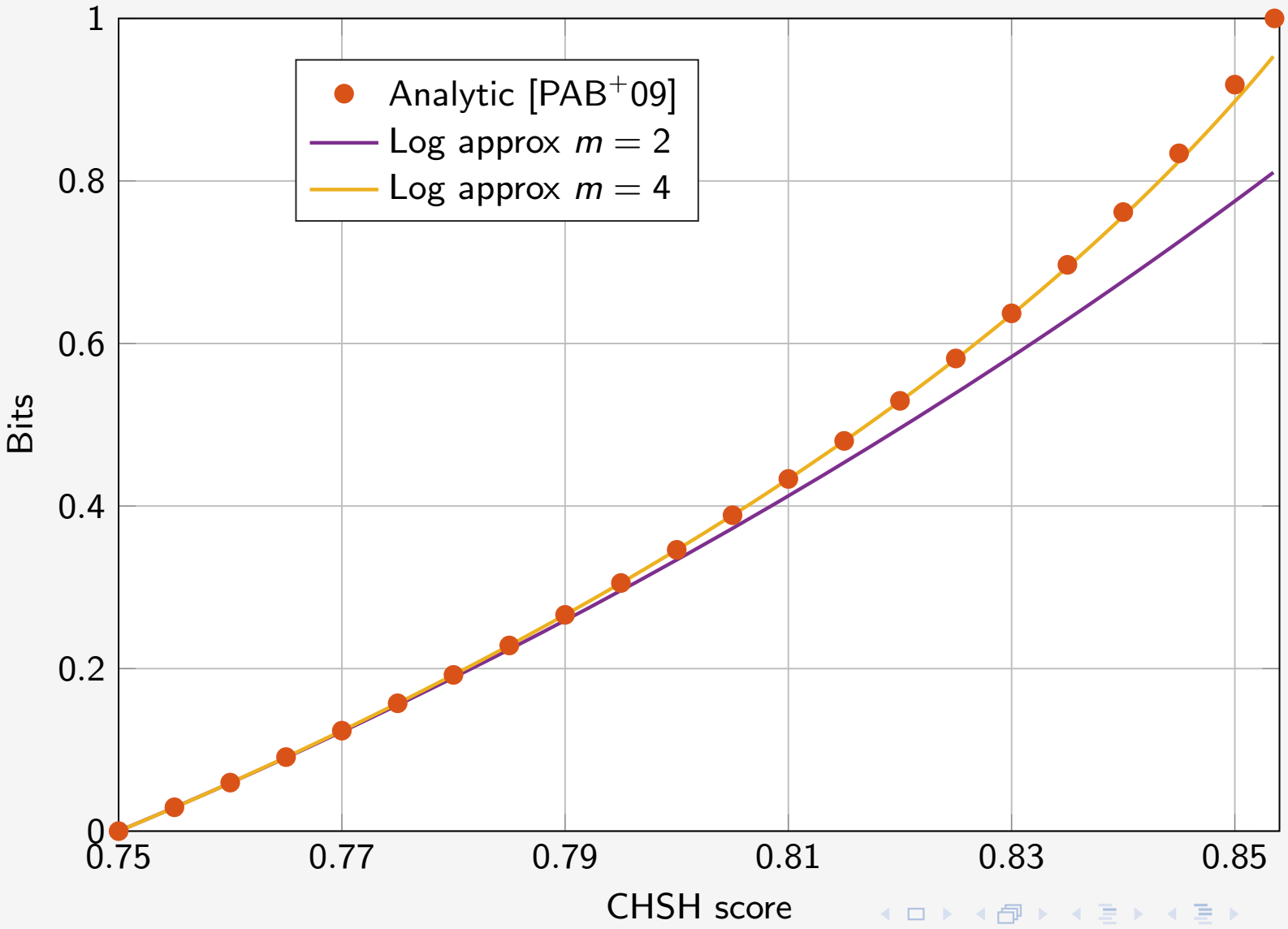

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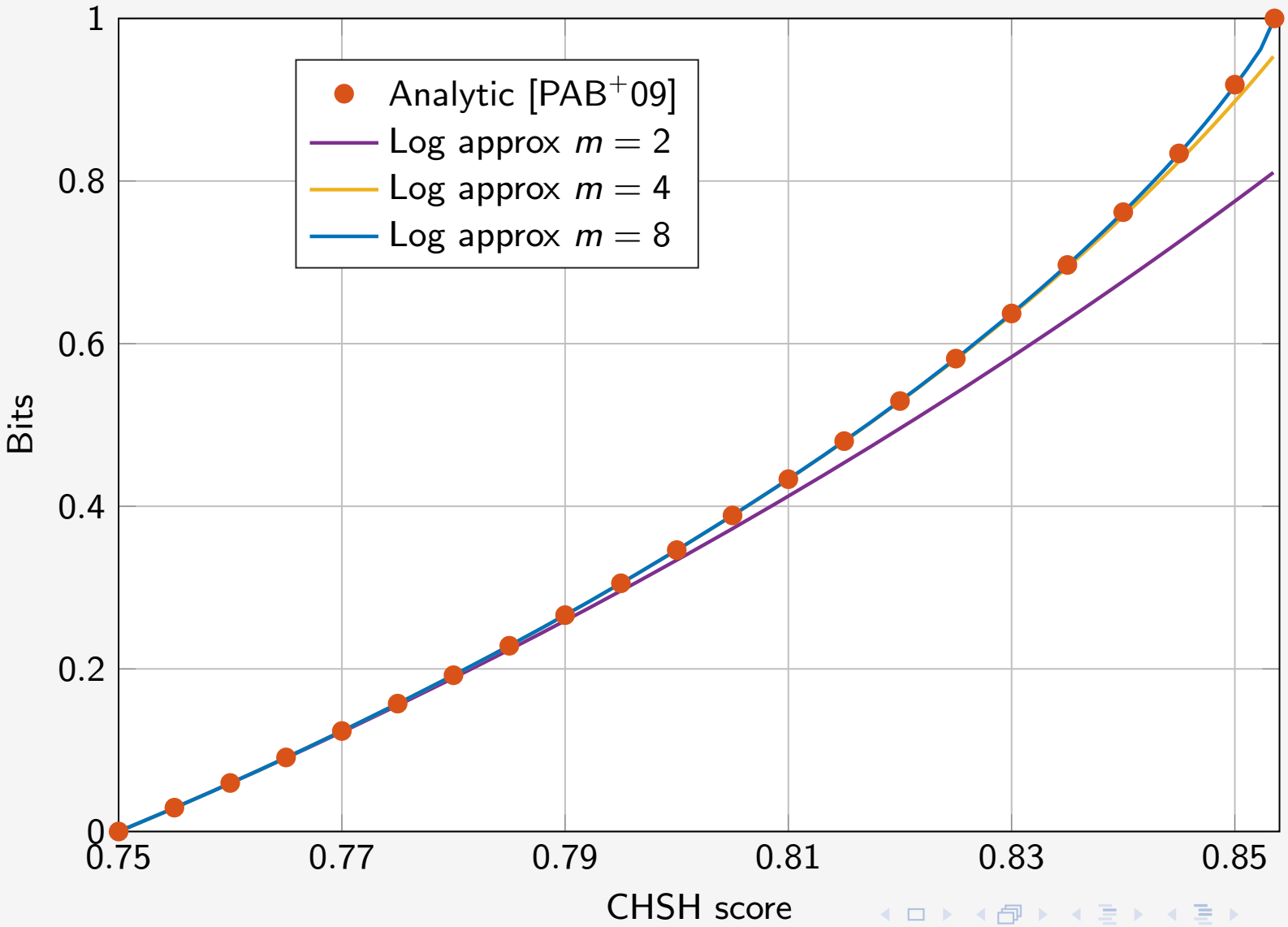
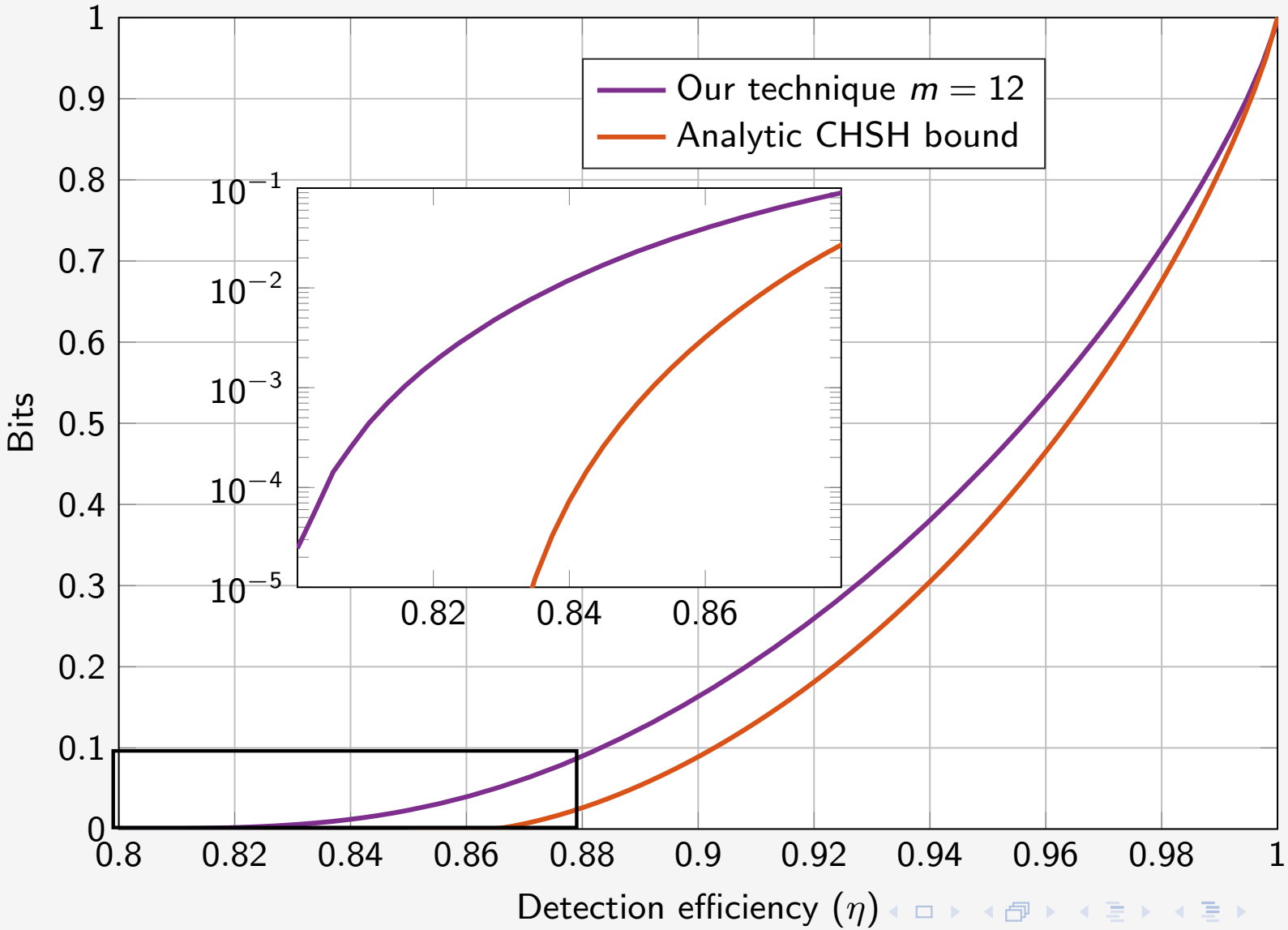


Results II – Improved DIQKD rates

Bounding inf $H(A|X=0,Q_E) - H(A|X=0,Y=2,B)$



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- SDP lower bounds via NPA hierarchy!

Results - Werner state squashed entanglement

Consider a two-qubit Werner state

$$\rho = p\frac{\Pi_{\mathrm{sym}}}{\mathrm{Tr}\left[\Pi_{\mathrm{sym}}\right]} + (1-p)\frac{\Pi_{\mathrm{asym}}}{\mathrm{Tr}\left[\Pi_{\mathrm{asym}}\right]}$$

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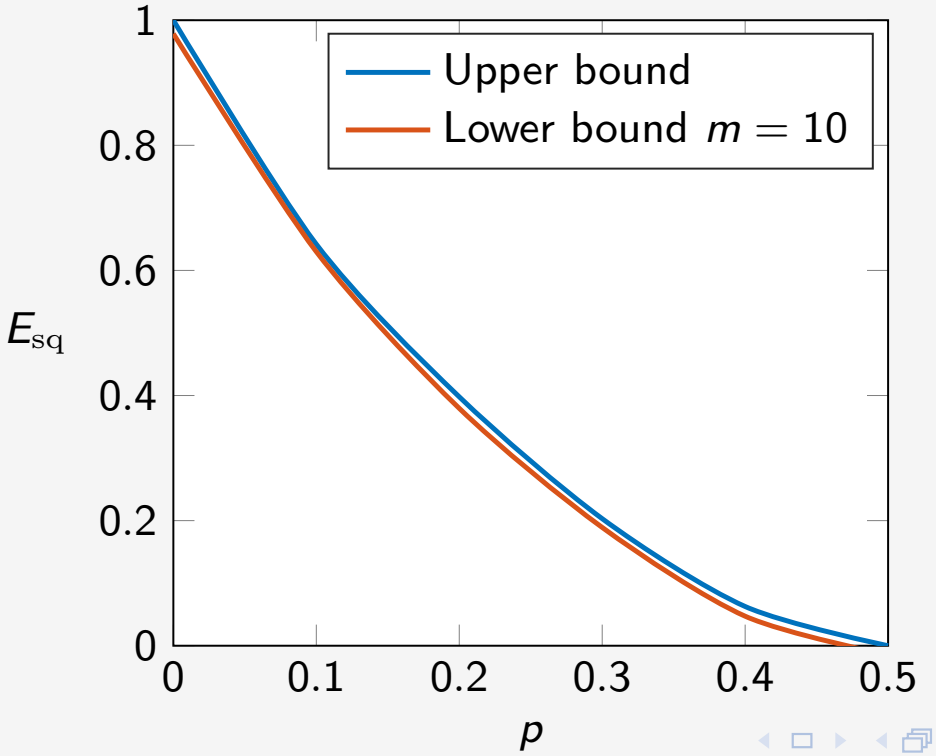
Using variational lower bounds and heuristic upper bounds we find

$$d_A = d_B = 2$$



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Bibliography

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Device-independent lower bounds

Fix some linear constraint(s) C on the joint probability distribution of the devices $p_{AB|XY}$. E.g.

$$\frac{1}{4}\sum_{xy=a\oplus b} p(ab|xy) \geq 0.8\,.$$

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Want to compute

$$r(C) = \inf \quad H(A|X=x^*, E)$$

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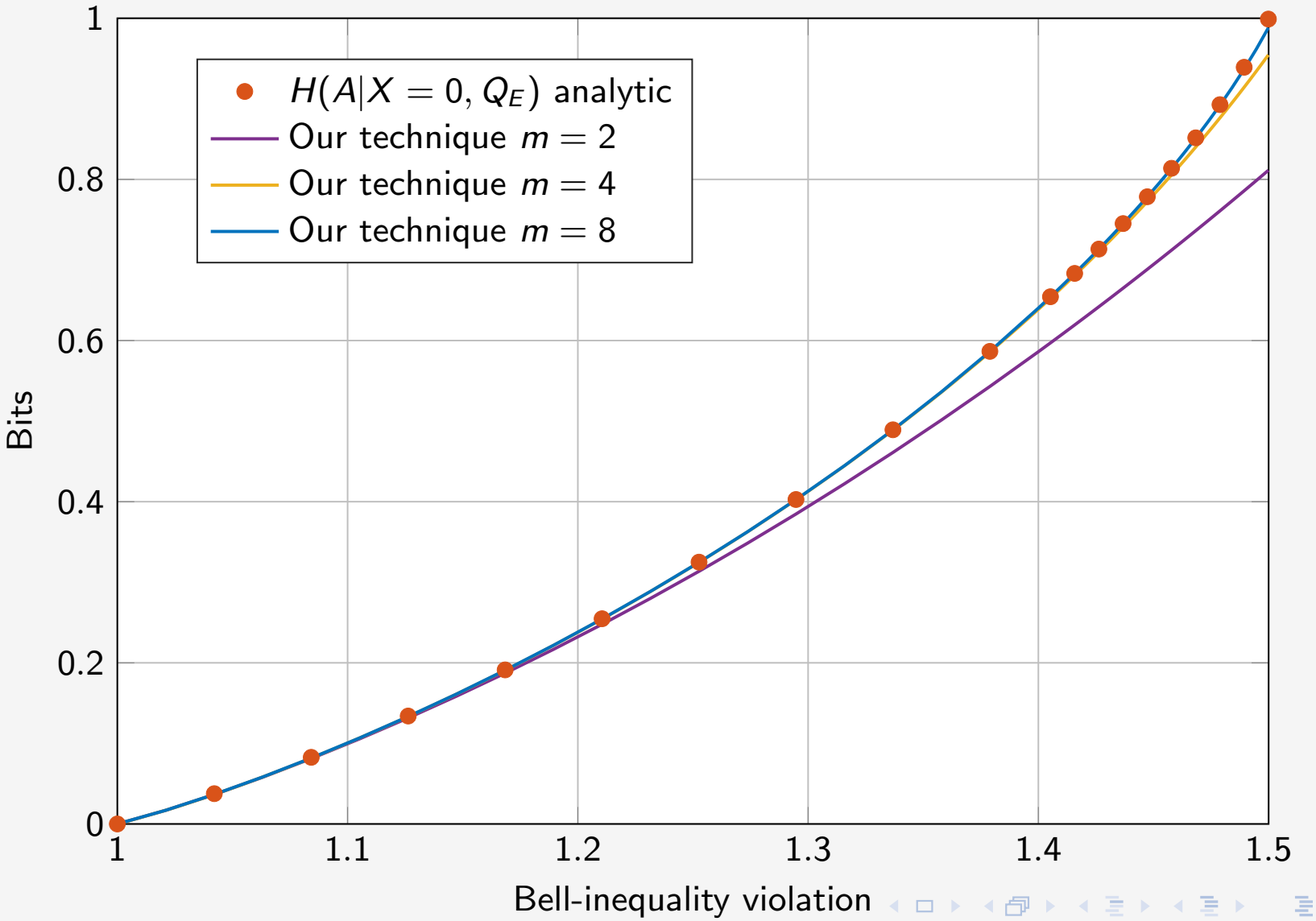
Difficult to solve
nonconvex / unbounded dimension

Bonus results – DICKA setting (Holz inequality)

Bounding inf $H(A|X=0,Q_E)$



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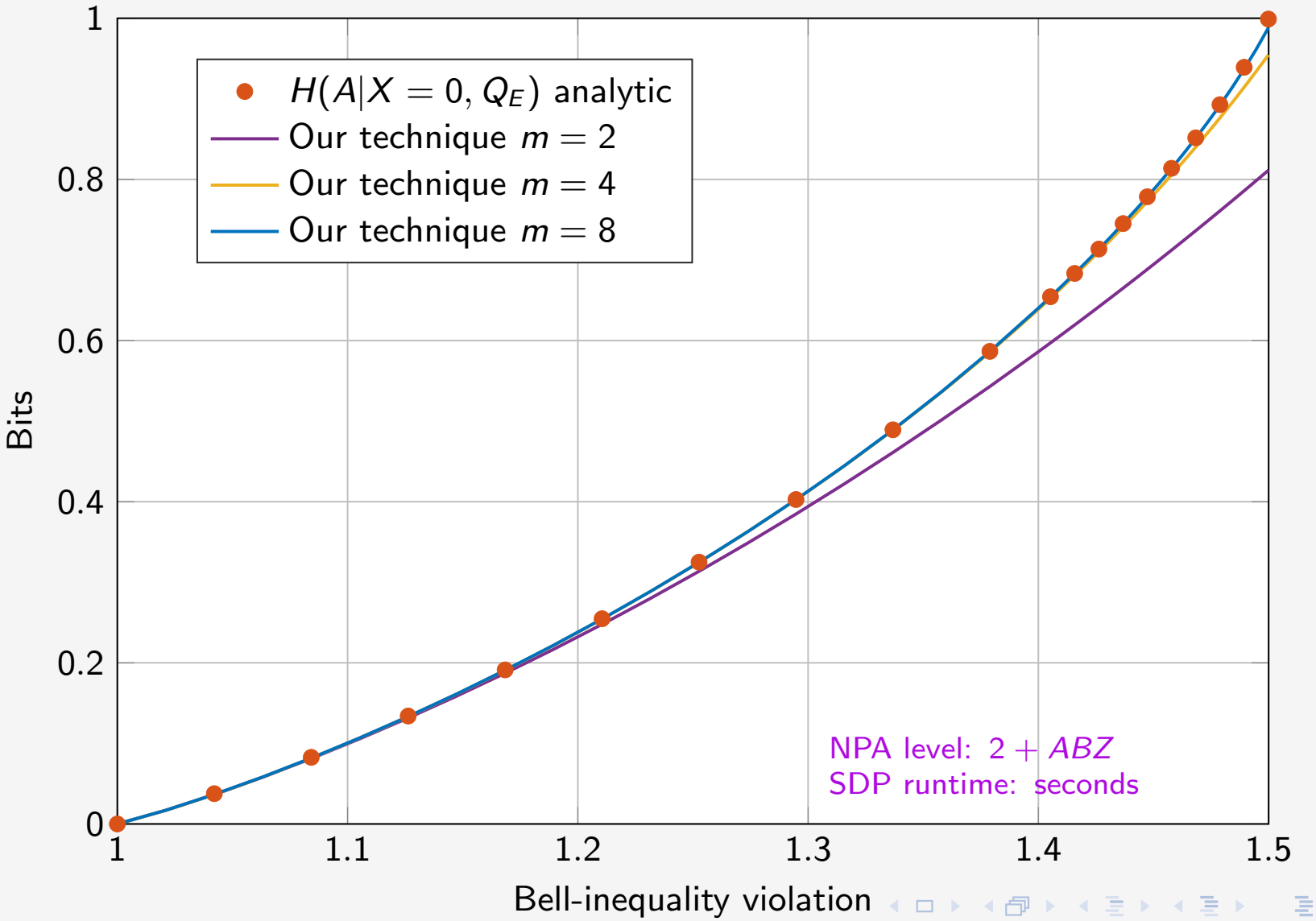
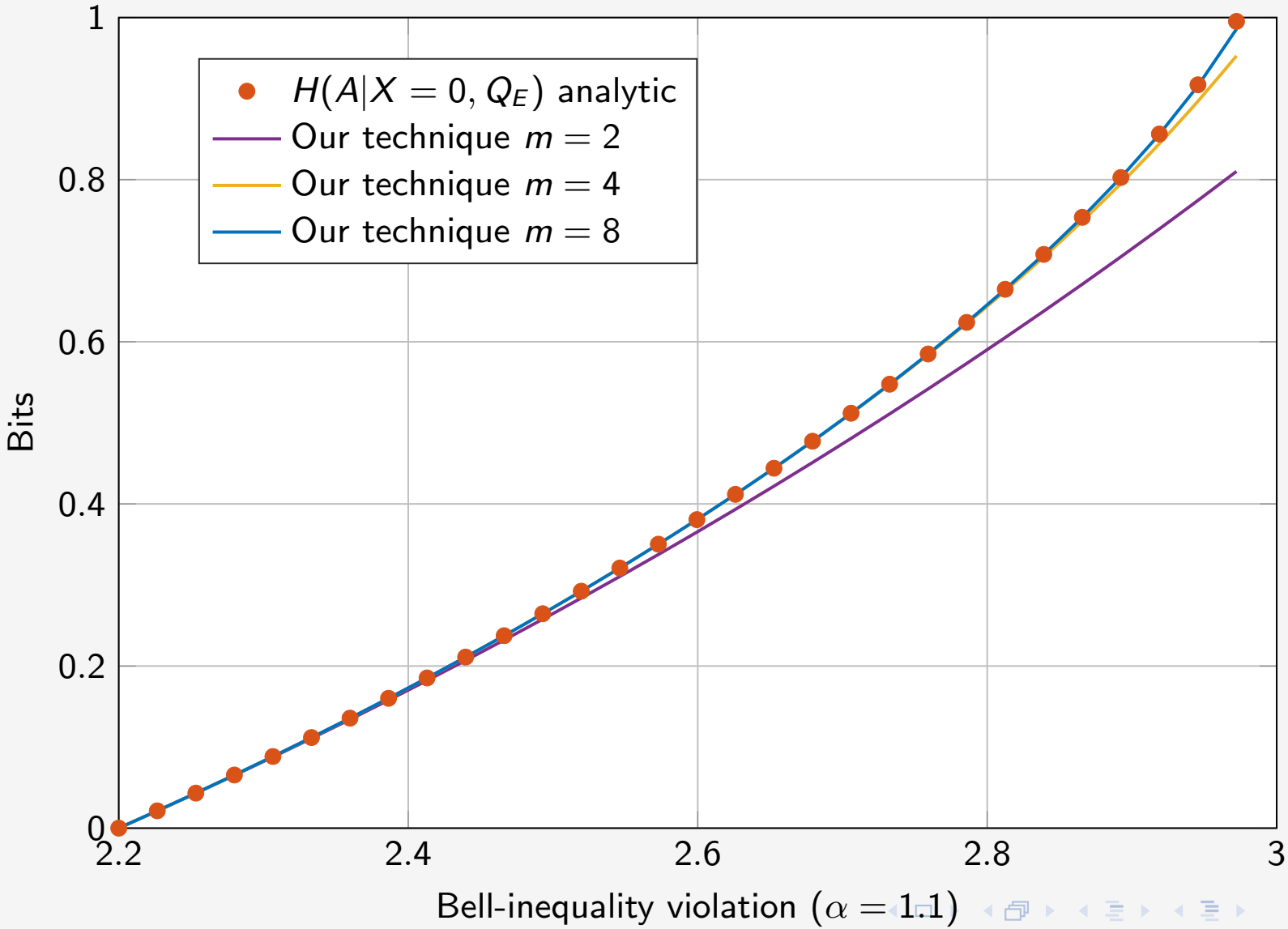


Bonus results – Generalized CHSH ($\alpha = 1.1$)

Bounding inf $H(A|X=0,Q_E)$ $\qquad B_\alpha = \alpha(\langle A_0B_0\rangle + \langle A_0B_1\rangle) + \langle A_1B_0\rangle - \langle A_1B_1\rangle$



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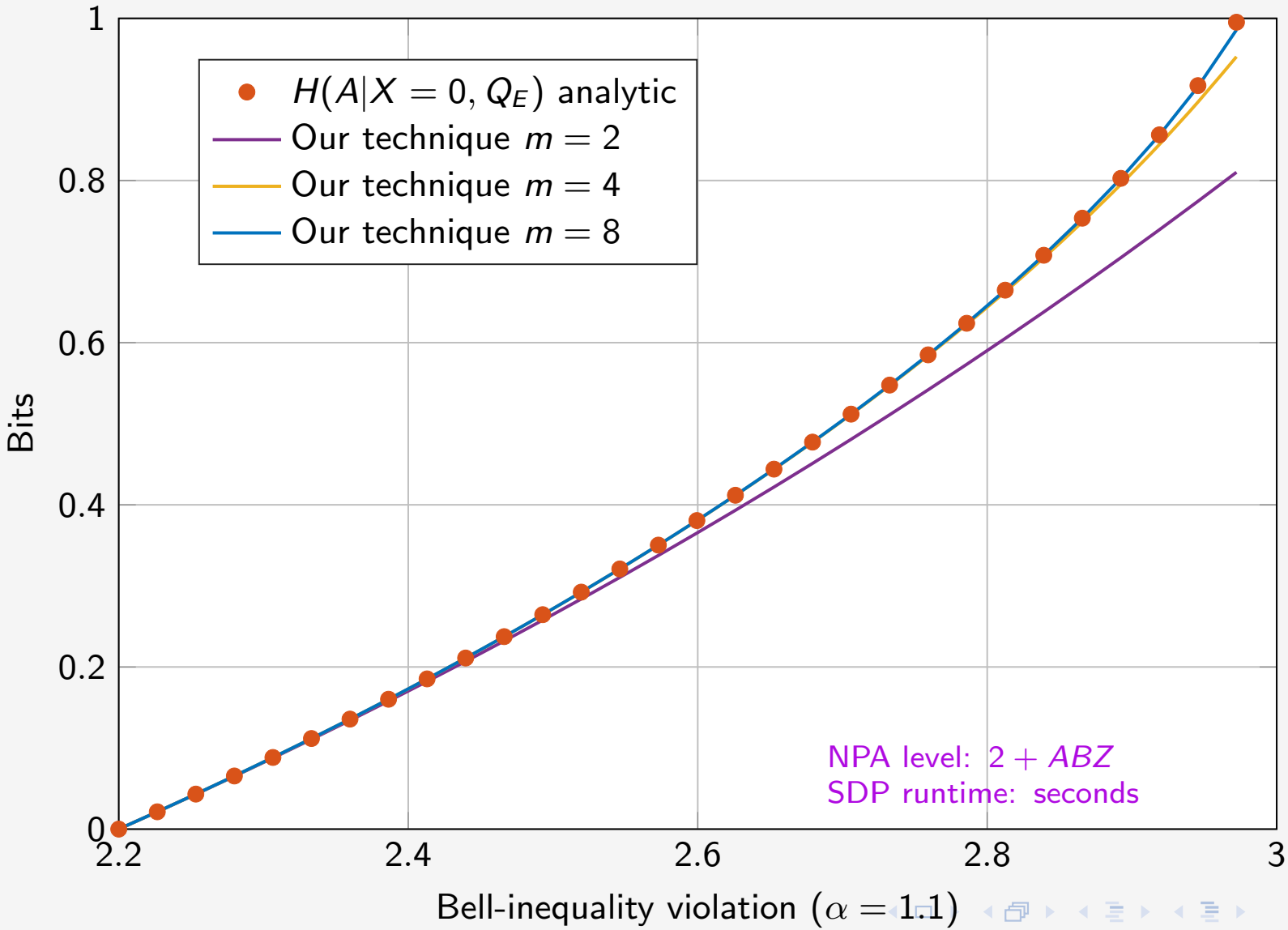

Bonus results – Generalized CHSH ($\alpha = 0.9$)

Bounding inf $H(A|X=0,Q_E)$ $\qquad$ $B_\alpha = \alpha(\langle A_0B_0\rangle + \langle A_0B_1\rangle) + \langle A_1B_0\rangle - \langle A_1B_1\rangle$