Device-independent lower bounds on the conditional von Neumann entropy

Peter Brown, Hamza Fawzi and Omar Fawzi

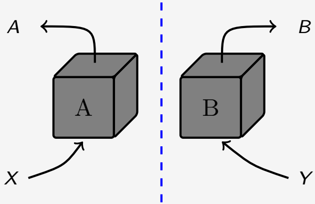
arXiv:2106.13692

Aug 31 2021

ENS DE LYON

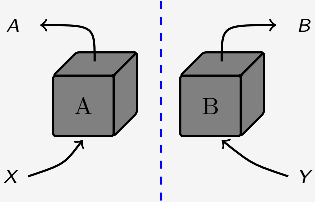
Motivation I

Bell-nonlocality



Motivation I

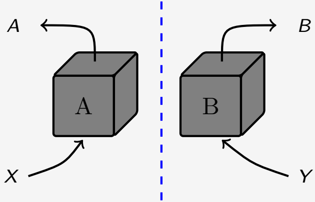
Bell-nonlocality



- Nonlocal correlations are inherently random.

Motivation I

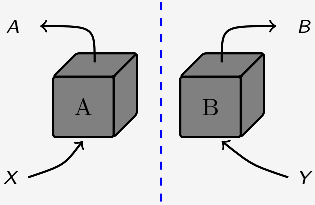
Bell-nonlocality



- Nonlocal correlations are inherently random.
- Foundation for randomness expansion / key-distribution protocols!

Motivation I

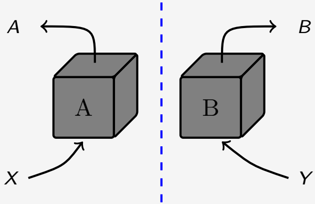
Bell-nonlocality



- Nonlocal correlations are inherently random.
- Foundation for randomness expansion / key-distribution protocols!
- Security and analysis relies on the *rate* (bits per round).

Motivation I

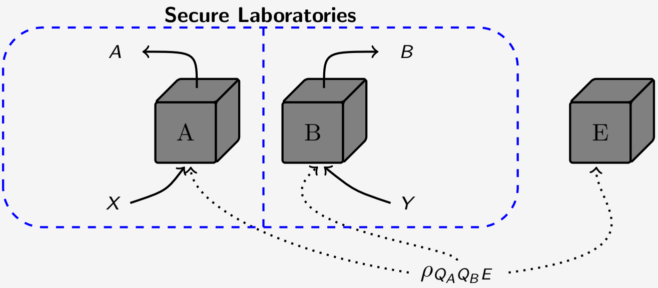
Bell-nonlocality



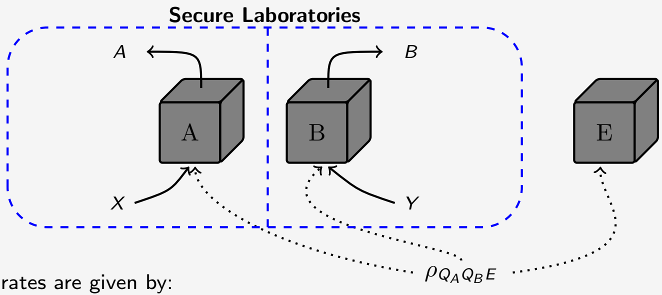
- Nonlocal correlations are inherently random.
- Foundation for randomness expansion / key-distribution protocols!
- Security and analysis relies on the *rate* (bits per round).

Main task of this work

Randomness generated per round



Randomness generated per round



Asymptotic rates are given by:

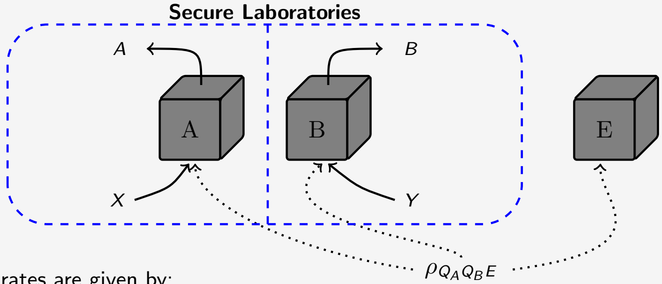
- **Randomness expansion**

$$H(AB|X = x^*, Y = y^*, E)$$

- **QKD**

$$H(A|X = x^*, E) - H(A|X = x^*, Y = y^*, B)$$

Randomness generated per round



Asymptotic rates are given by:

- **Randomness expansion**

$$H(AB|X = x^*, Y = y^*, E)$$

- **QKD**

$$H(A|X = x^*, E) - H(A|X = x^*, Y = y^*, B)$$

Want device-independent lower bounds

Device-independent lower bounds

Fix some linear constraint(s) C on the joint probability distribution of the devices $p_{AB|XY}$. E.g.

$$\frac{1}{4}\sum_{xy=a\oplus b} p(ab|xy) \geq 0.8\,.$$

Device-independent lower bounds

Fix some linear constraint(s) C on the joint probability distribution of the devices $p_{AB|XY}$. E.g.

$$\frac{1}{4}\sum_{xy=a\oplus b} p(ab|xy) \geq 0.8\,.$$

A **strategy** for C is a tuple $(Q_A Q_B Q_E, \rho, \{\{M_{a|x}\}_a\}_x, \{\{N_{b|y}\}_b\}_y)$ such that

$$p(ab|xy) = \mathrm{Tr}\left[\rho(M_{a|x} \otimes N_{b|y} \otimes I_E)\right]$$

satisfies the constraints in C.

Device-independent lower bounds

Fix some linear constraint(s) C on the joint probability distribution of the devices $p_{AB|XY}$. E.g.

$$\frac{1}{4}\sum_{xy=a\oplus b} p(ab|xy) \geq 0.8\,.$$

A **strategy** for C is a tuple $(Q_A Q_B Q_E, \rho, \{\{M_{a|x}\}_a\}_x, \{\{N_{b|y}\}_b\}_y)$ such that

$$p(ab|xy) = \mathrm{Tr}\left[\rho(M_{a|x}\otimes N_{b|y}\otimes I_E)\right]$$

satisfies the constraints in C.

Through the post measurement state

$$\rho_{AQ_E} = \sum_a |a\rangle\langle a| \otimes \mathrm{Tr}_{Q_A Q_B}\left[(M_{a|x^*}\otimes I)\rho\right] \longrightarrow H(A|X=x^*, Q_E)$$

Device-independent lower bounds

Fix some linear constraint(s) C on the joint probability distribution of the devices $p_{AB|XY}$. E.g.

$$\frac{1}{4} \sum_{xy=a\oplus b} p(ab|xy) \geq 0.8 \,.$$

A **strategy** for C is a tuple $(Q_A Q_B Q_E, \rho, \{\{M_{a|x}\}_a\}_x, \{\{N_{b|y}\}_b\}_y)$ such that

$$p(ab|xy) = \mathrm{Tr}\left[\rho(M_{a|x} \otimes N_{b|y} \otimes I_E)\right]$$

satisfies the constraints in C.

Through the post measurement state

$$\rho_{AQ_E} = \sum_a |a\rangle\langle a| \otimes \mathrm{Tr}_{Q_A Q_B}\left[(M_{a|x^*} \otimes I)\rho\right] \longrightarrow H(A|X = x^*, Q_E)$$

DI bounds

Want to compute

$$r(C) = \inf \quad H(A|X = x^*, E)$$

where inf over all strategies compatible with C.

Device-independent lower bounds

Fix some linear constraint(s) C on the joint probability distribution of the devices $p_{AB|XY}$. E.g.

$$\frac{1}{4}\sum_{xy=a\oplus b} p(ab|xy) \geq 0.8\,.$$

A **strategy** for C is a tuple $(Q_A Q_B Q_E, \rho, \{\{M_{a|x}\}_a\}_x, \{\{N_{b|y}\}_b\}_y)$ such that

$$p(ab|xy) = \mathrm{Tr}\left[\rho(M_{a|x} \otimes N_{b|y} \otimes I_E)\right]$$

satisfies the constraints in C.

Through the post measurement state

$$\rho_{AQ_E} = \sum_a |a\rangle\langle a| \otimes \mathrm{Tr}_{Q_A Q_B}\left[(M_{a|x^*} \otimes I)\rho\right] \longrightarrow H(A|X = x^*, Q_E)$$

DI bounds

Want to compute

$$r(C) = \inf \quad H(A|X = x^*, E)$$

where inf over all strategies compatible with C.

Difficult to solve
nonconvex / unbounded dimension

Previous works

Approaches

- Analytical bounds [PAB+09, GMKB21, MPW21]
  - Reduce to qubits and solve explicitly
  - tight bounds / restricted scope

Previous works

Approaches

- Analytical bounds [PAB+09, GMKB21, MPW21]
  - Reduce to qubits and solve explicitly
  - tight bounds / restricted scope

$\inf \mathrm{Tr}\left[\rho p(Z)\right]$

- The min-entropy $H_{\min}$
  - Write as a noncommutative polynomial optimization problem (NCPOP) and apply NPA.
  - easy to compute / poor bounds

Previous works

Approaches

- Analytical bounds [PAB$^+$09, GMKB21, MPW21]
  - Reduce to qubits and solve explicitly
  - tight bounds / restricted scope

$\inf \mathrm{Tr}\,[\rho p(Z)]$

- The min-entropy $H_{\min}$
  - Write as a noncommutative polynomial optimization problem (NCPOP) and apply NPA.
  - easy to compute / poor bounds
- Recent works [TSG$^+$19, BFF21]
  - Different lower bounding NCPOPs.
  - Better than $H_{\min}$ / room for improvement

Previous works

Approaches

- Analytical bounds [PAB+09, GMKB21, MPW21]
  - Reduce to qubits and solve explicitly
  - tight bounds / restricted scope

$\inf \mathrm{Tr}\left[\rho p(Z)\right]$

- The min-entropy $H_{\min}$
  - Write as a noncommutative polynomial optimization problem (NCPOP) and apply NPA.
  - easy to compute / poor bounds
- Recent works [TSG+19, BFF21]
  - Different lower bounding NCPOPs.
  - Better than $H_{\min}$ / room for improvement
- Our new approach
  - Define a sequence

$$H_m(\rho) = \inf_{Z_1,\ldots,Z_m \in B(H)} \mathrm{Tr}\left[\rho \, q(Z_1,\ldots,Z_m)\right] \tag{1}$$

    such that $H_m \leq H$ and $H_m \to H$ as $m \to \infty$.
  - close to optimal / more efficient / wider scope

Previous works

Approaches

- Analytical bounds [PAB+09, GMKB21, MPW21]
  - Reduce to qubits and solve explicitly
  - tight bounds / restricted scope
- The min-entropy $H_{\min}$
  - Write as a noncommutative polynomial optimization problem (NCPOP) and apply NPA.
  - easy to compute / poor bounds

inf $\mathrm{Tr}\left[\rho p(Z)\right]$

- Recent works [TSG+19, BFF21]
  - Different lower bounding NCPOPs.
  - Better than $H_{\min}$ / room for improvement

Numerical approaches can complement analytical ones

- Our new approach
  - Define a sequence

$$H_m(\rho) = \inf_{Z_1,\ldots,Z_m \in B(H)} \mathrm{Tr}\left[\rho\, q(Z_1,\ldots,Z_m)\right] \quad (1)$$

    such that $H_m \leq H$ and $H_m \to H$ as $m \to \infty$.
  - close to optimal / more efficient / wider scope

Generalization: relative entropy bounds

We actually work with the relative entropy

$$D(\rho\|\sigma) = \mathrm{Tr}\left[\rho(\log \rho - \log \sigma)\right].$$

Generalization: relative entropy bounds

We actually work with the relative entropy

$$D(\rho\|\sigma) = \mathrm{Tr}\left[\rho(\log\rho - \log\sigma)\right].$$

Can use it for conditional entropy

$$H(A|B) = -D(\rho_{AB}\|I_A \otimes \rho_B).$$

Generalization: relative entropy bounds

We actually work with the relative entropy

$$D(\rho\|\sigma) = \mathrm{Tr}\left[\rho(\log\rho - \log\sigma)\right].$$

Can use it for conditional entropy

$$H(A|B) = -D(\rho_{AB}\|I_A \otimes \rho_B).$$

The goal

Derive something of the form

$$D(\rho\|\sigma) \leq \sum_{i=1}^{m} \inf_{Z} \mathrm{Tr}\left[\rho p_i(Z)\right] + \mathrm{Tr}\left[\sigma q_i(Z)\right]$$

with p_i and q_i some polynomials and with the RHS converging as $m \to \infty$.

Derivation overview

1. Gauss-Radau approximation of the logarithm

$$\ln(x) = \int_0^1 \frac{x-1}{t(x-1)+1} \mathrm{d}t \geq \sum_{i=1}^{m} w_i f_{t_i}(x)$$

where $f_t(x) = \frac{x-1}{t(x-1)+1}$ (RHS converges as $m \to \infty$).

Derivation overview

1. Gauss-Radau approximation of the logarithm

$$\ln(x) = \int_0^1 \frac{x-1}{t(x-1)+1}\mathrm{d}t \geq \sum_{i=1}^{m} w_i f_{t_i}(x)$$

where $f_t(x) = \frac{x-1}{t(x-1)+1}$ (RHS converges as $m \to \infty$).

2. Apply approximation to logarithm in $D(\rho\|\sigma)$

$$D(\rho\|\sigma) \leq \sum_{i=1}^{m} \frac{w_i}{\ln 2} D_{f_{t_i}}(\rho\|\sigma).$$

Derivation overview

1. Gauss-Radau approximation of the logarithm

$$\ln(x) = \int_0^1 \frac{x-1}{t(x-1)+1} \mathrm{d}t \geq \sum_{i=1}^{m} w_i f_{t_i}(x)$$

   where $f_t(x) = \frac{x-1}{t(x-1)+1}$ (RHS converges as $m \to \infty$).

2. Apply approximation to logarithm in $D(\rho\|\sigma)$

$$D(\rho\|\sigma) \leq \sum_{i=1}^{m} \frac{w_i}{\ln 2} D_{f_{t_i}}(\rho\|\sigma).$$

3. Each $D_{f_t}(\rho\|\sigma)$ admits a variational form

$$D_{f_t}(\rho\|\sigma) = \frac{1}{t} \inf_{Z \in B(H)} \{\mathrm{Tr}\,[\rho(I + Z + Z^* + (1-t)Z^*Z)] + t\mathrm{Tr}\,[\sigma Z Z^*]\}$$

Derivation overview

1. Gauss-Radau approximation of the logarithm

$$\ln(x) = \int_0^1 \frac{x-1}{t(x-1)+1}\mathrm{d}t \geq \sum_{i=1}^{m} w_i f_{t_i}(x)$$

where $f_t(x) = \frac{x-1}{t(x-1)+1}$ (RHS converges as $m \to \infty$).

2. Apply approximation to logarithm in $D(\rho\|\sigma)$

$$D(\rho\|\sigma) \leq \sum_{i=1}^{m} \frac{w_i}{\ln 2} D_{f_{t_i}}(\rho\|\sigma).$$

3. Each $D_{f_t}(\rho\|\sigma)$ admits a variational form

$$D_{f_t}(\rho\|\sigma) = \frac{1}{t} \inf_{Z \in B(H)} \{\mathrm{Tr}\left[\rho(I + Z + Z^* + (1-t)Z^*Z)\right] + t\mathrm{Tr}\left[\sigma Z Z^*\right]\}$$

Result

$$D(\rho\|\sigma) \leq \sum_{i=1}^{m} \frac{w_i}{t_i \ln 2} \inf_{Z \in B(H)} \{\mathrm{Tr}\left[\rho(I + Z + Z^* + (1-t_i)Z^*Z)\right] + t_i\mathrm{Tr}\left[\sigma Z Z^*\right]\}$$

and RHS converges as $m \to \infty$.

Lower bound on $H(A|X = x^*, Q_E)$

$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

$$c_m + \inf_{\textit{strategies}} \sum_{i=1}^{m-1} \frac{w_i}{t_i \ln 2} \sum_a \mathrm{Tr}\left[\rho_{Q_A Q_E}(M_{a|x^*} \otimes (Z_{a,i} + Z_{a,i}^* + (1-t_i) Z_{a,i}^* Z_{a,i}) + t_i Z_{a,i} Z_{a,i}^*)\right] .$$

Lower bound on $H(A|X = x^*, Q_E)$

$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

$$c_m + \inf_{\text{strategies}} \sum_{i=1}^{m-1} \frac{w_i}{t_i \ln 2} \sum_a \mathrm{Tr}\left[\rho_{Q_A Q_E}(M_{a|x^*} \otimes (Z_{a,i} + Z_{a,i}^* + (1-t_i) Z_{a,i}^* Z_{a,i}) + t_i Z_{a,i} Z_{a,i}^*)\right] .$$

Remarks

- Can now be easily relaxed to an NCPOP and solved using NPA [PNA10].

Lower bound on $H(A|X = x^*, Q_E)$

$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

$$c_m + \inf_{\text{strategies}} \sum_{i=1}^{m-1} \frac{w_i}{t_i \ln 2} \sum_a \mathrm{Tr}\left[\rho_{Q_A Q_E}(M_{a|x^*} \otimes (Z_{a,i} + Z_{a,i}^* + (1-t_i) Z_{a,i}^* Z_{a,i}) + t_i Z_{a,i} Z_{a,i}^*)\right].$$

Drop $\otimes$ and impose $[M, Z] = 0$.

Remarks

- Can now be easily relaxed to an NCPOP and solved using NPA [PNA10].

Lower bound on $H(A|X = x^*, Q_E)$

$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

$$c_m + \inf_{strategies} \sum_{i=1}^{m-1} \frac{w_i}{t_i \ln 2} \sum_a \mathrm{Tr}\left[\rho_{Q_A Q_E}(M_{a|x^*} \otimes (Z_{a,i} + Z^*_{a,i} + (1-t_i) Z^*_{a,i} Z_{a,i}) + t_i Z_{a,i} Z^*_{a,i})\right] .$$

Drop $\otimes$ and impose $[M, Z] = 0$.

Remarks

- Can now be easily relaxed to an NCPOP and solved using NPA [PNA10].
- NPA hierarchy converges as $\|Z\|$ can be bounded.

Lower bound on $H(A|X = x^*, Q_E)$

$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

$$c_m + \inf_{\text{strategies}} \sum_{i=1}^{m-1} \frac{w_i}{t_i \ln 2} \sum_a \mathrm{Tr}\left[\rho_{Q_A Q_E}(M_{a|x^*} \otimes (Z_{a,i} + Z_{a,i}^* + (1-t_i) Z_{a,i}^* Z_{a,i}) + t_i Z_{a,i} Z_{a,i}^*)\right] .$$

Drop $\otimes$ and impose $[M, Z] = 0$.

Remarks

- Can now be easily relaxed to an NCPOP and solved using NPA [PNA10].
- NPA hierarchy converges as $\|Z\|$ can be bounded.
- Similar results for $H(AB|X = x, Y = y, Q_E)$ or $H(A|XQ_E)$ and others.

Lower bound on $H(A|X = x^*, Q_E)$

$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

$$c_m + \inf_{strategies} \sum_{i=1}^{m-1} \frac{w_i}{t_i \ln 2} \sum_a \mathrm{Tr}\left[\rho_{Q_A Q_E}(M_{a|x^*} \otimes (Z_{a,i} + Z_{a,i}^* + (1-t_i) Z_{a,i}^* Z_{a,i}) + t_i Z_{a,i} Z_{a,i}^*)\right] .$$

Drop $\otimes$ and impose $[M, Z] = 0$.

Remarks

- Can now be easily relaxed to an NCPOP and solved using NPA [PNA10].
- NPA hierarchy converges as $\|Z\|$ can be bounded.
- Similar results for $H(AB|X = x, Y = y, Q_E)$ or $H(A|XQ_E)$ and others.

Caveats

- Number of operators grows with m.

Lower bound on $H(A|X = x^*, Q_E)$

$H(A|B) = -D(\rho_{AB} \| I_A \otimes \rho_B)$

Theorem

The rate $\inf H(A|X = x^*, Q_E)$ *is never smaller than*

$$c_m + \inf_{\text{strategies}} \sum_{i=1}^{m-1} \frac{w_i}{t_i \ln 2} \sum_a \mathrm{Tr}\left[\rho_{Q_A Q_E} (M_{a|x^*} \otimes (Z_{a,i} + Z_{a,i}^* + (1 - t_i) Z_{a,i}^* Z_{a,i}) + t_i Z_{a,i} Z_{a,i}^*)\right] .$$

Drop $\otimes$ and impose $[M, Z] = 0$.

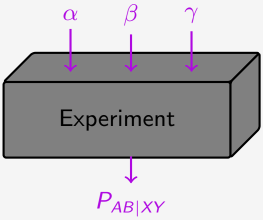
Remarks

- Can now be easily relaxed to an NCPOP and solved using NPA [PNA10].
- NPA hierarchy converges as $\|Z\|$ can be bounded.
- Similar results for $H(AB|X = x, Y = y, Q_E)$ or $H(A|XQ_E)$ and others.

Caveats

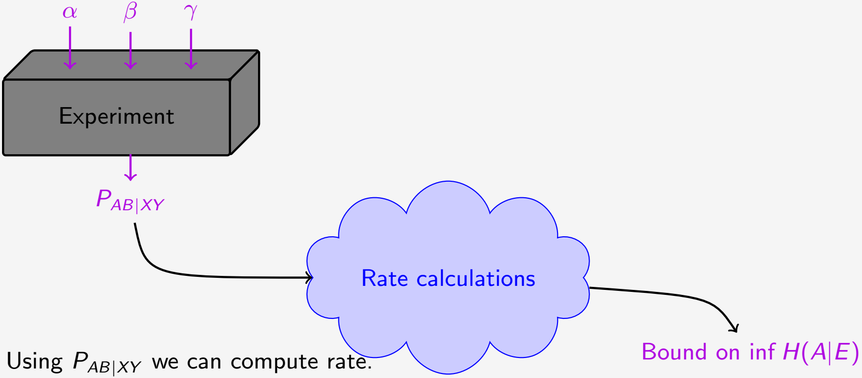
- Number of operators grows with m. Use $\inf \sum_i \cdots \geq \sum_i \inf \ldots$ to stop such scaling

Optimizing experiments



- Distribution depends on parameters – $P_{\alpha,\beta,\gamma}(a, b|x, y)$

Optimizing experiments



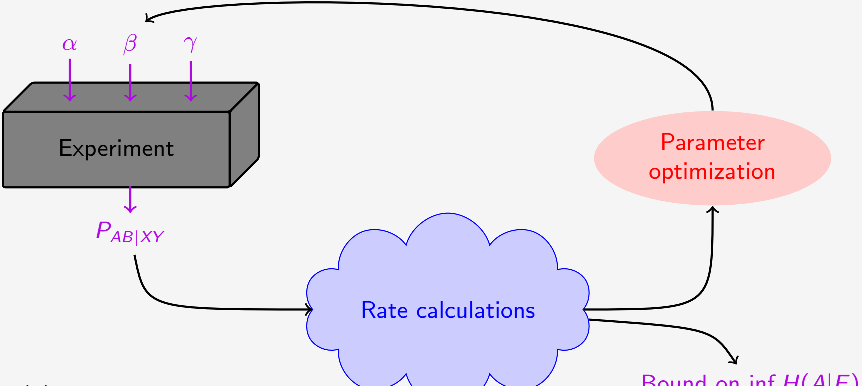
- Using $P_{AB|XY}$ we can compute rate.

$$\begin{aligned} p(b) = \inf_{X} \quad & \mathrm{Tr}\left[CX\right] \\ \text{s.t.} \quad & \mathrm{Tr}\left[F_i X\right] = b_i \qquad \forall i \\ & X \geq 0 \end{aligned}$$

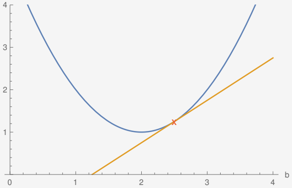
$$\begin{aligned} d(b) = \sup_{\lambda, Y} \quad & \sum_i \lambda_i b_i \\ \text{s.t.} \quad & C - \sum_i \lambda_i F_i - Y \geq 0 \\ & Y \geq 0 \end{aligned}$$

$g(b) = \sum_i \lambda_i b_i$

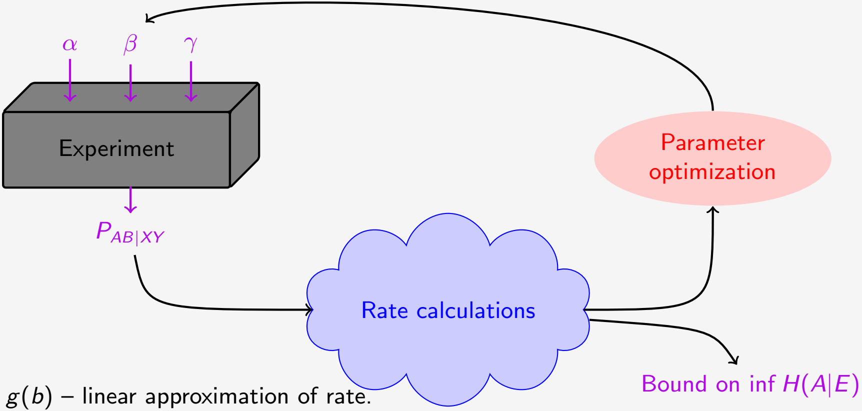
Optimizing experiments



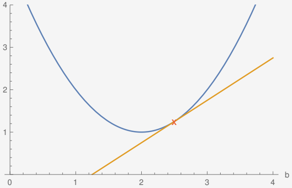
- $g(b)$ – linear approximation of rate.



Optimizing experiments

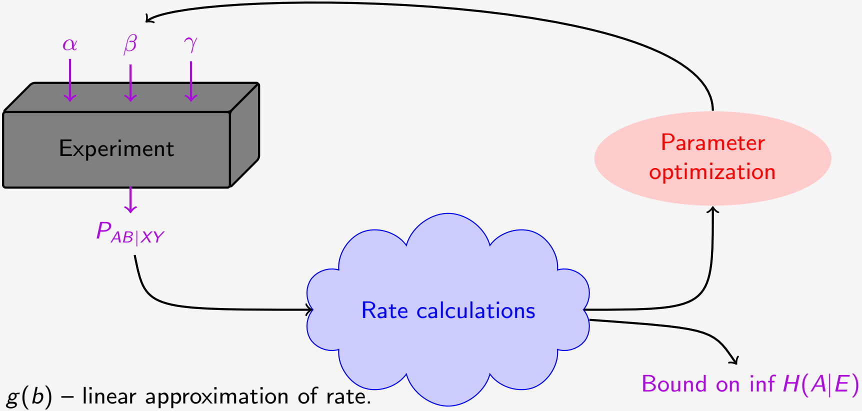


- $g(b)$ – linear approximation of rate.



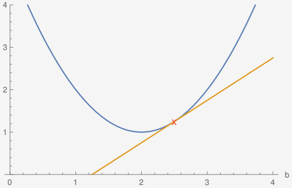
Maximize $g(b(\alpha, \beta, \gamma))$!

Optimizing experiments



- $g(b)$ – linear approximation of rate.

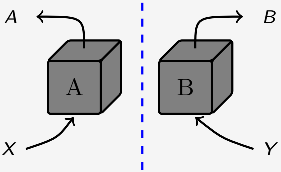
Effectively a min-tradeoff function!



Maximize $g(b(\alpha, \beta, \gamma))$!

Results

- Applied our method to compute rates for DIRNG and DIQKD.



Results

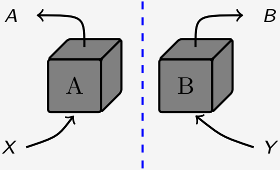
- Applied our method to compute rates for DIRNG and DIQKD.

- Experimental parameters: $(\theta, a_0, a_1, \ldots, b_0, b_1, \ldots)$ where

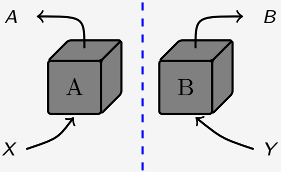
$$|\psi\rangle_{Q_A Q_B} = \cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$$

$$M_{0|x} = \frac{1}{2}(I + \cos(a_x)\sigma_z + \sin(a_x)\sigma_x)$$

$$N_{0|y} = \frac{1}{2}(I + \cos(b_y)\sigma_z + \sin(b_y)\sigma_x)$$



Results

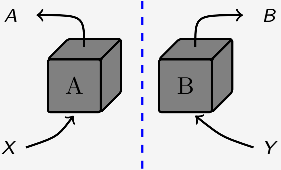


- Applied our method to compute rates for DIRNG and DIQKD.
- Experimental parameters: $(\theta, a_0, a_1, \ldots, b_0, b_1, \ldots)$ where

$$\begin{aligned} |\psi\rangle_{Q_A Q_B} &= \cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle \\ M_{0|x} &= \frac{1}{2}(I + \cos(a_x)\sigma_z + \sin(a_x)\sigma_x) \\ N_{0|y} &= \frac{1}{2}(I + \cos(b_y)\sigma_z + \sin(b_y)\sigma_x) \end{aligned}$$

- Looked at different constraint sets C:
  - CHSH score
    $$\tfrac{1}{4}\sum_{xy=a\oplus b} p(ab|xy) \geq \omega$$
  - Full distribution
    $$p(ab|xy) = c_{abxy} \qquad \forall (a,b,x,y)$$

Results



- Applied our method to compute rates for DIRNG and DIQKD.
- Experimental parameters: $(\theta, a_0, a_1, \ldots, b_0, b_1, \ldots)$ where

$$|\psi\rangle_{Q_A Q_B} = \cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$$
$$M_{0|x} = \frac{1}{2}(I + \cos(a_x)\sigma_z + \sin(a_x)\sigma_x)$$
$$N_{0|y} = \frac{1}{2}(I + \cos(b_y)\sigma_z + \sin(b_y)\sigma_x)$$

- Looked at different constraint sets C:
  - CHSH score

$$\tfrac{1}{4} \sum_{xy = a \oplus b} p(ab|xy) \geq \omega$$

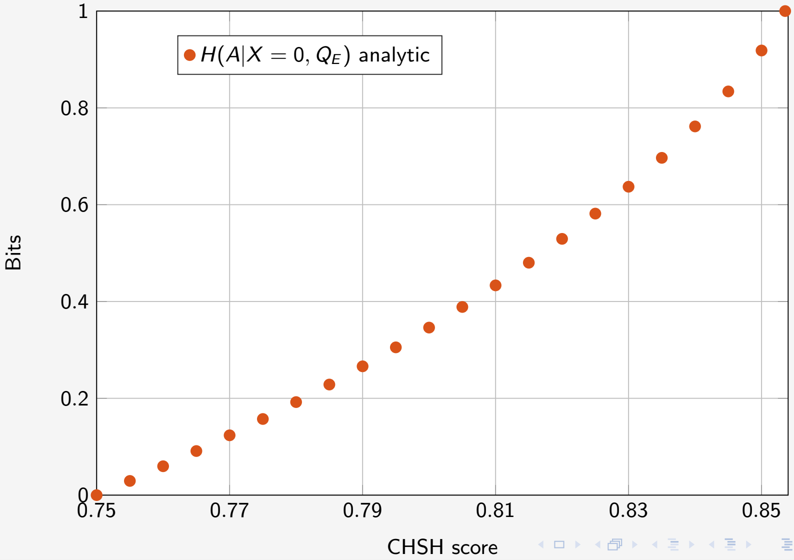
  - Full distribution

$$p(ab|xy) = c_{abxy} \qquad \forall (a, b, x, y)$$

- Investigated *detection efficiency* noise model.
  - Independent probability $\eta \in [0, 1]$ that each device *succeeds*.
  - Device failures recorded as a particular outcome.

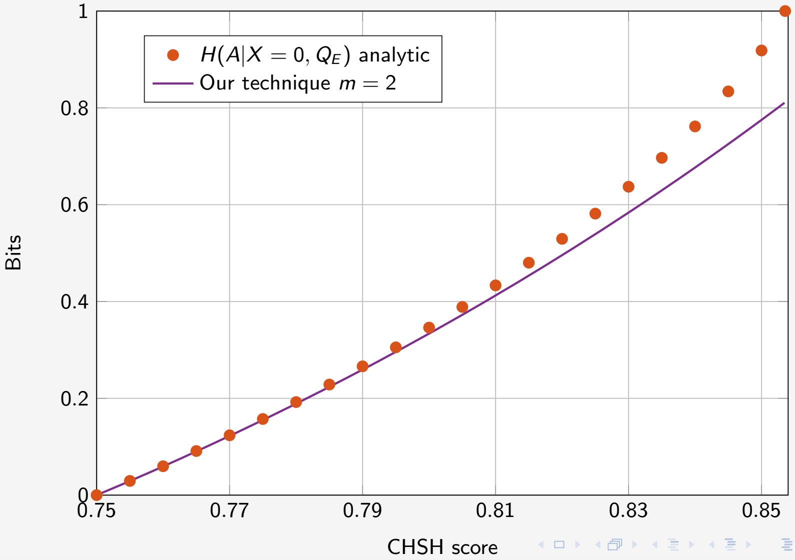
Results I – Recovering tight bounds for the CHSH game

Bounding inf $H(A|X=0, Q_E)$



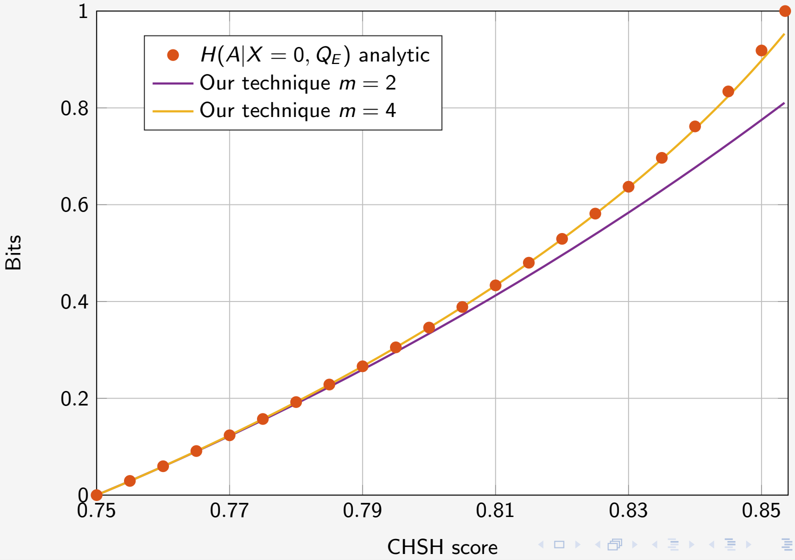
Results I – Recovering tight bounds for the CHSH game

Bounding $\inf H(A|X=0, Q_E)$



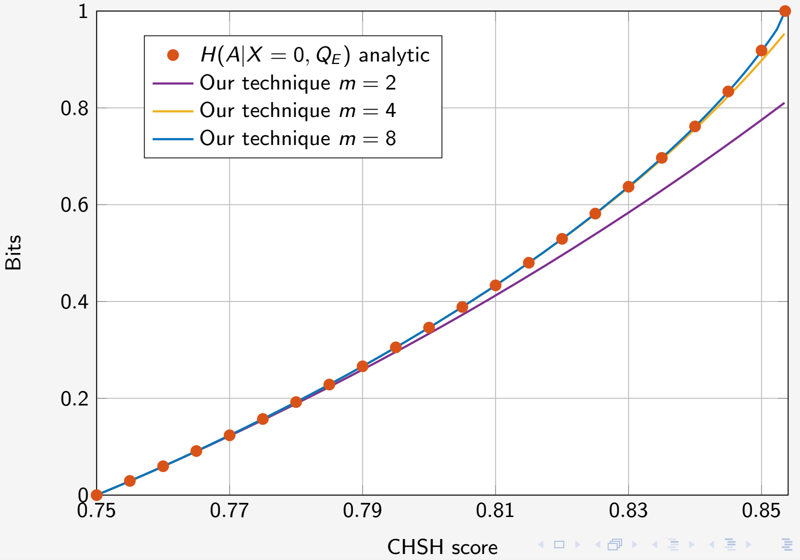
Results I – Recovering tight bounds for the CHSH game

Bounding $\inf H(A|X=0, Q_E)$



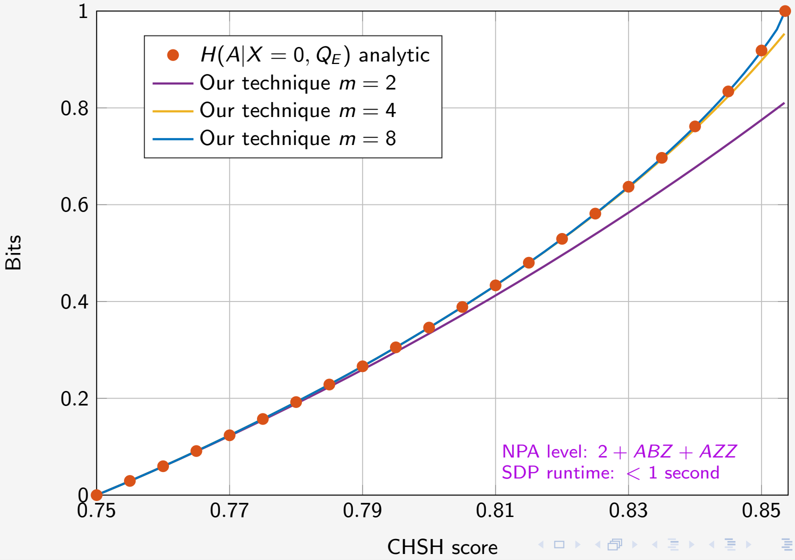
Results I – Recovering tight bounds for the CHSH game

Bounding inf $H(A|X = 0, Q_E)$



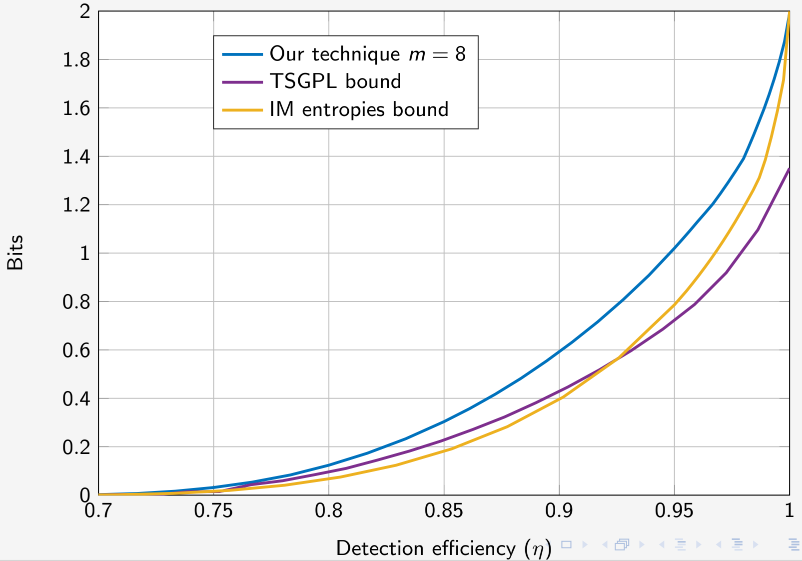
Results I – Recovering tight bounds for the CHSH game

Bounding $\inf H(A|X=0, Q_E)$



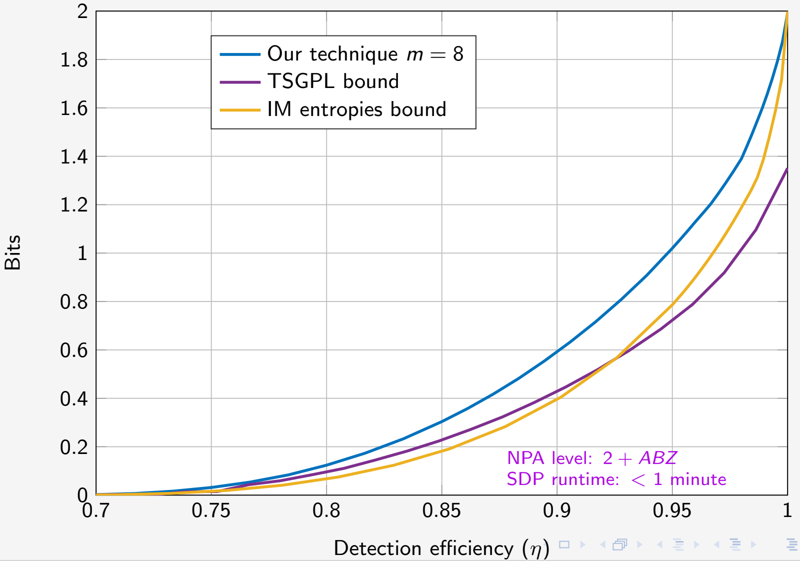
Results II – Improved randomness expansion rates

Bounding $\inf H(AB|X=0, Y=0, Q_E)$



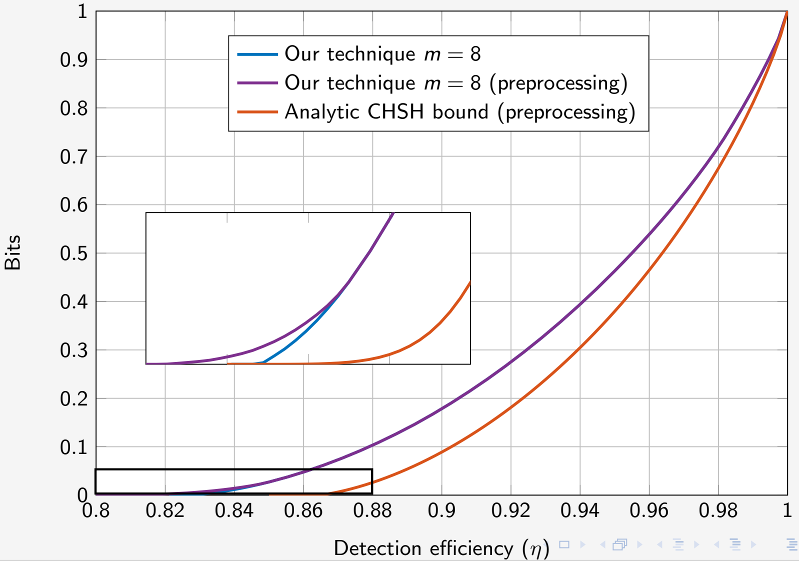
Results II – Improved randomness expansion rates

Bounding inf $H(AB|X = 0, Y = 0, Q_E)$



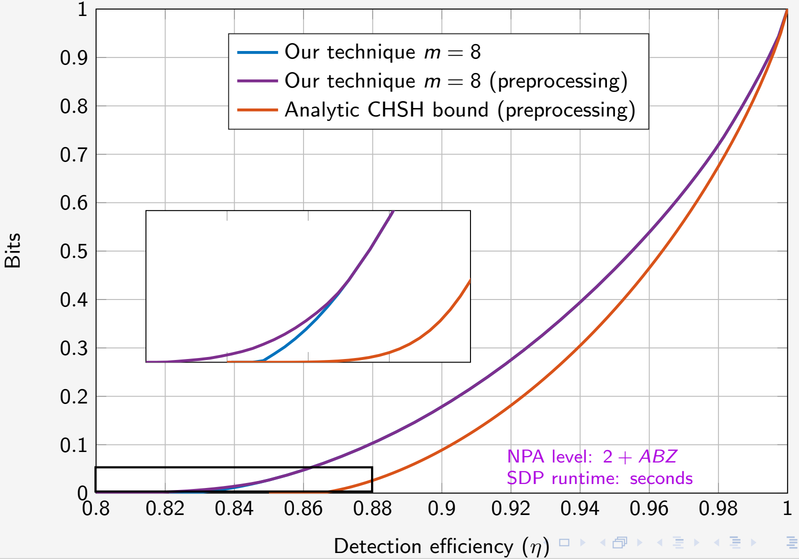
Results III – Improved DIQKD rates

Bounding $\inf H(A|X=0,Q_E) - H(A|X=0,Y=2,B)$



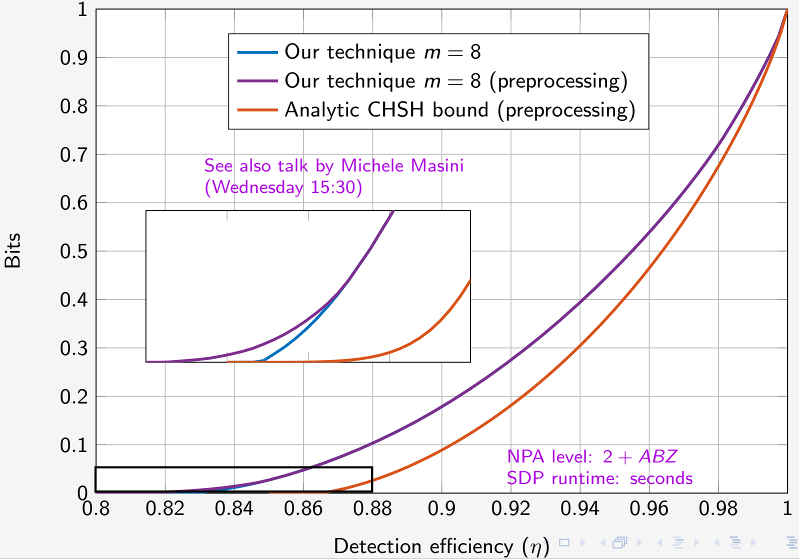
Results III – Improved DIQKD rates

Bounding $\inf H(A|X=0, Q_E) - H(A|X=0, Y=2, B)$



Results III – Improved DIQKD rates

Bounding $\inf H(A|X=0, Q_E) - H(A|X=0, Y=2, B)$



Conclusion

Summary

- New general method to compute rates of DI protocols.

Conclusion

Summary

- New general method to compute rates of DI protocols.
- Convergent (in a sense) – observe practical convergence also.

Conclusion

Summary

- New general method to compute rates of DI protocols.
- Convergent (in a sense) – observe practical convergence also.
- Outperforms all previous methods (+ faster)

Conclusion

Summary

- New general method to compute rates of DI protocols.
- Convergent (in a sense) – observe practical convergence also.
- Outperforms all previous methods (+ faster)
- Applies to infinite dimensional systems and can be used directly with EAT to prove security.

Conclusion

Summary

- New general method to compute rates of DI protocols.
- Convergent (in a sense) – observe practical convergence also.
- Outperforms all previous methods (+ faster)
- Applies to infinite dimensional systems and can be used directly with EAT to prove security.

Outlook

- Better understand convergence? (commuting operator vs tensor product).

Conclusion

Summary

- New general method to compute rates of DI protocols.
- Convergent (in a sense) – observe practical convergence also.
- Outperforms all previous methods (+ faster)
- Applies to infinite dimensional systems and can be used directly with EAT to prove security.

Outlook

- Better understand convergence? (commuting operator vs tensor product).
- Is DIQKD feasible now? (Better experimental model / finite size analysis)

Conclusion

Summary

- New general method to compute rates of DI protocols.
- Convergent (in a sense) – observe practical convergence also.
- Outperforms all previous methods (+ faster)
- Applies to infinite dimensional systems and can be used directly with EAT to prove security.

Outlook

- Better understand convergence? (commuting operator vs tensor product).
- Is DIQKD feasible now? (Better experimental model / finite size analysis)
- Beyond DIQKD?

Bibliography

Peter Brown, Hamza Fawzi, and Omar Fawzi.
Computing conditional entropies for quantum correlations.
Nature communications, 12(1):1–12, 2021.

Federico Grasselli, Gláucia Murta, Hermann Kampermann, and Dagmar Bruß.
In preparation.
2021.

Timo Holz, Hermann Kampermann, and Dagmar Bruß.
Genuine multipartite bell inequality for device-independent conference key agreement.
Physical Review Research, 2(2):023251, 2020.

Michele Masini, Stefano Pironio, and Erik Woodhead.
Simple and practical diqkd security analysis via bb84-type uncertainty relations and pauli correlation constraints.
arXiv preprint arXiv:2107.08894, 2021.

Stefano Pironio, Antonio Acín, Nicolas Brunner, Nicolas Gisin, Serge Massar, and Valerio Scarani.
Device-independent quantum key distribution secure against collective attacks.
New Journal of Physics, 11(4):045021, 2009.

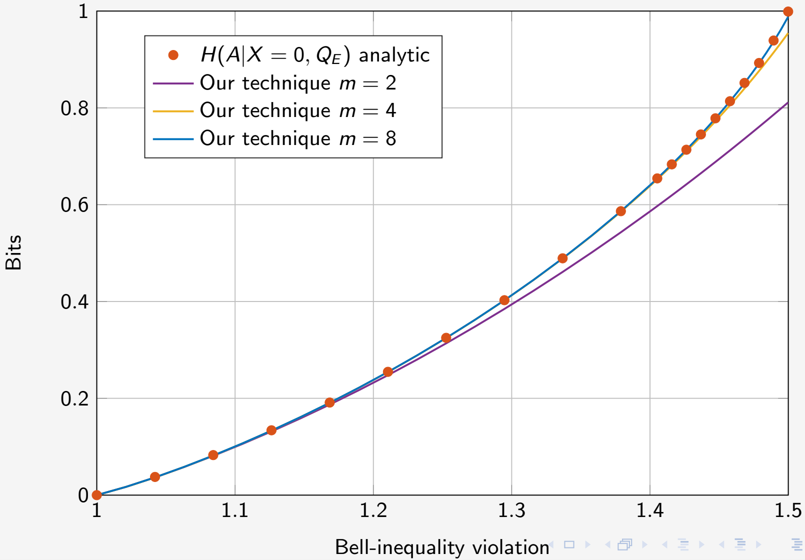
Stefano Pironio, Miguel Navascués, and Antonio Acín.
Convergent relaxations of polynomial optimization problems with noncommuting variables.
SIAM Journal on Optimization, 20(5):2157–2180, 2010.

Ernest Y-Z Tan, René Schwonnek, Koon Tong Goh, Ignatius William Primaatmaja, and Charles C-W Lim.
Computing secure key rates for quantum key distribution with untrusted devices.
e-print arXiv:1908.11372, 2019.

Erik Woodhead, Antonio Acín, and Stefano Pironio.
Device-independent quantum key distribution with asymmetric chsh inequalities.
Quantum, 5:443, 2021.

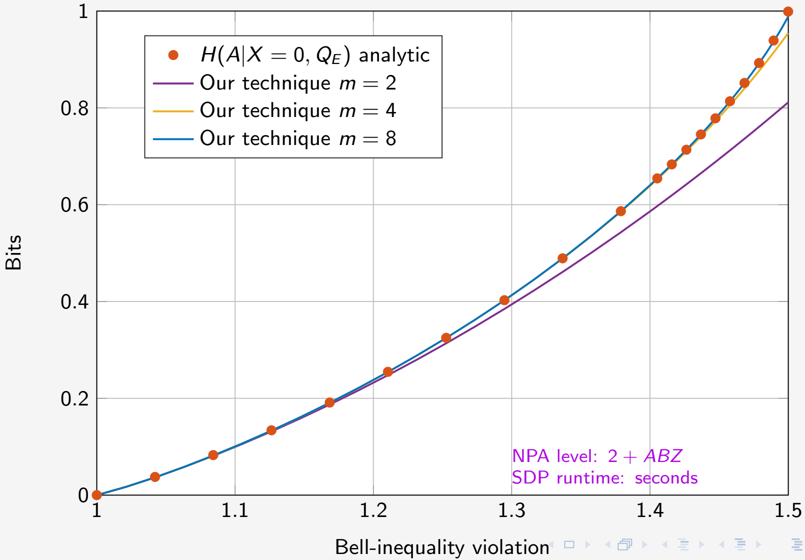
Bonus results – DICKA setting (Holz inequality [HKB20])

Bounding inf $H(A|X=0, Q_E)$



Bonus results – DICKA setting (Holz inequality [HKB20])

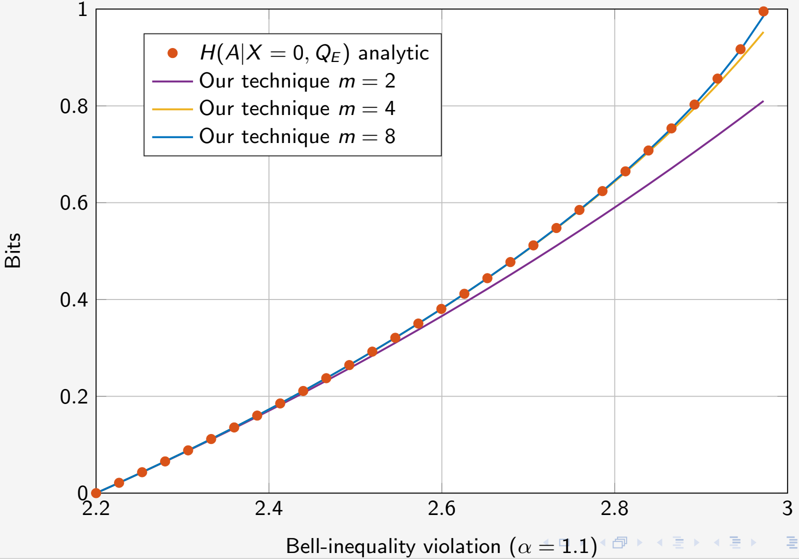
Bounding inf $H(A|X=0, Q_E)$



Bonus results – Generalized CHSH [WAP21] ($\alpha = 1.1$)

Bounding $\inf H(A|X=0, Q_E)$

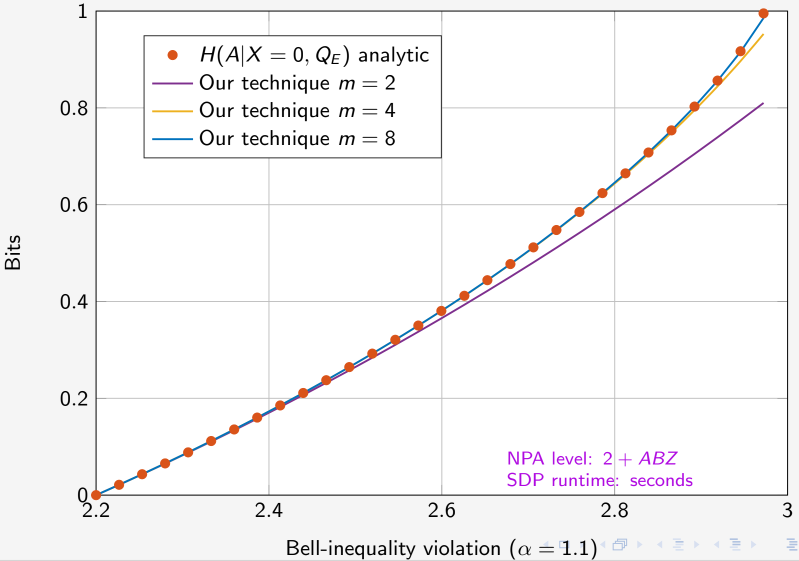
$$B_\alpha = \alpha(\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle) + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$



Bonus results – Generalized CHSH [WAP21] ($\alpha = 1.1$)

Bounding $\inf H(A|X=0, Q_E)$

$$B_\alpha = \alpha(\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle) + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$



Bonus results – Generalized CHSH [WAP21] ($\alpha = 0.9$)

Bounding $\inf H(A|X=0, Q_E)$ $\qquad$ $B_\alpha = \alpha(\langle A_0 B_0\rangle + \langle A_0 B_1\rangle) + \langle A_1 B_0\rangle - \langle A_1 B_1\rangle$