Arbitrarily many independent observers can share the nonlocality of a single maximally entangled qubit pair

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joint work with Roger Colbeck[2]

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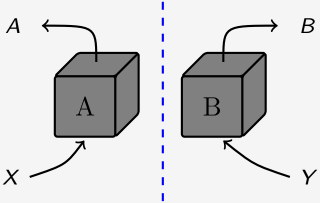
Motivation I

Bell-nonlocality



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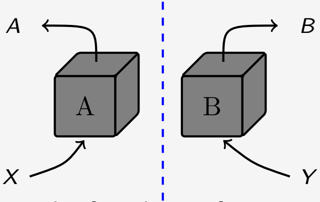

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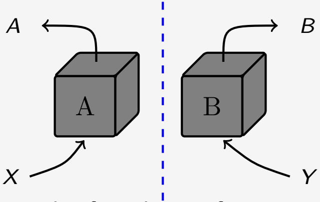

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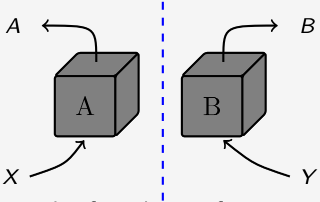
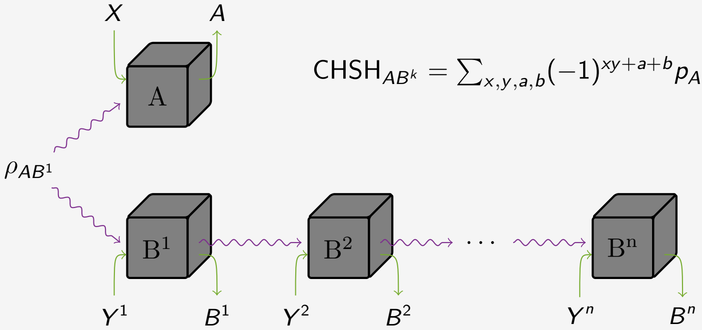


- Nonlocal correlations are the foundation for many device independent protocols
- There are measurements that do not destroy the entanglement between the two halves of the state.
- Can we use this remaining entanglement to generate more nonlocal correlations?

The scenario

We focus on the following scenario introduced in [SGGP15].



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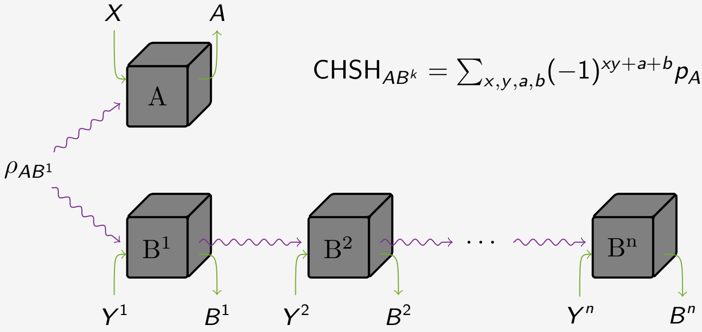

$$\mathrm{CHSH}_{AB^k} = \textstyle\sum_{x,y,a,b}(-1)^{xy+a+b}p_{AB^k}(a,b|x,y)$$

- All inputs/outputs are binary - inputs chosen uniformly.
- Quantum state passed to next Bob (but not the input/output information)

$$\rho_{AB^n} = \frac{1}{2}\sum_{b_{n-1}y_{n-1}}(I \otimes F^{1/2}_{b_{n-1}|y_{n-1}})\rho_{AB^{n-1}}(I \otimes F^{1/2}_{b_{n-1}|y_{n-1}})$$

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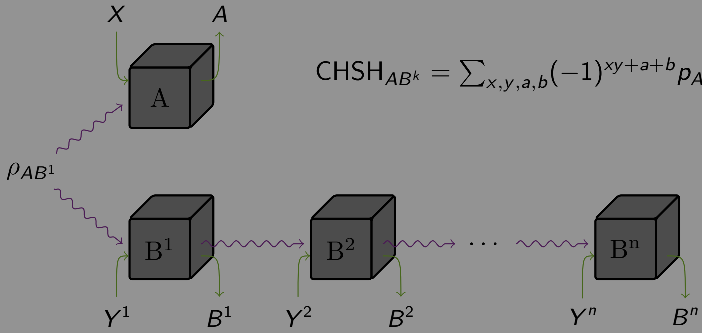

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- All
- Qu ... ation)

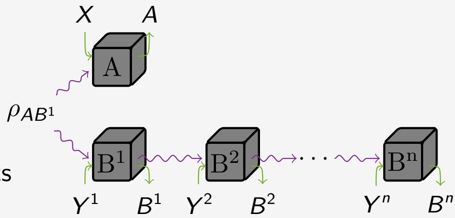
Main question

Suppose Alice and Bob^1 share the state ρ_{AB^1}. Then what is the maximum number of Bob's that can achieve an expected CHSH violation with Alice?

$b_{n-1}y_{n-1}$

The scenario II

Previous works:



- [SGGP15]: 2-qubit maximally entangled state + heavily biased inputs $\implies$ unbounded #-violations.
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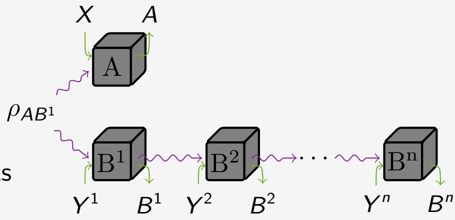

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In this work we show this statement is in general false.

- We construct an explicit measurement strategy for any $n \in \mathbb{N}$ such that n Bobs can violate.
- Also extend this strategy to a larger class of states including any pure two-qubit state.

The strategy

We consider qubit POVMs of the form $\{M, I - M\}$ where the effects are given by $M = I/2 + \gamma(\cos(\varphi)\sigma_z + \sin(\varphi)\sigma_x)/2$, where $\varphi \in [-\pi, \pi]$ and $\gamma \in [0, 1]$ is the *sharpness*.

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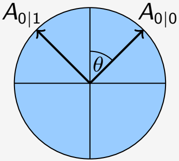
Alice's measurements: For $\theta \in (0, \pi/4]$

$$A_{0|0} = \frac{I + \cos(\theta)\sigma_z + \sin(\theta)\sigma_x}{2}$$

and

$$A_{0|1} = \frac{I + \cos(\theta)\sigma_z - \sin(\theta)\sigma_x}{2}.$$



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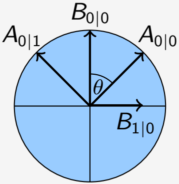

Bob^k's measurements: For $\gamma_k \in (0, 1)$

$$B^k_{0|0} = \frac{I + \sigma_z}{2}$$

and

$$B^k_{0|1} = \frac{I + \gamma_k \sigma_x}{2}.$$

The strategy II

If Alice and Bob[1] start with the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ then

$$\mathrm{CHSH}_{AB^k} = 2^{2-k}\left(\gamma_k \sin(\theta) + \cos(\theta)\prod_{j=1}^{k-1}\left(1+\sqrt{1-\gamma_j^2}\right)\right).$$

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Theorem

For any $n \in \mathbb{N}$ there exists $\theta \in (0, \pi/4]$ and $(\gamma_1, \ldots, \gamma_n) \in (0,1)^n$ such that $\mathrm{CHSH}_{AB^k} > 2$ for all $1 \leq k \leq n$.

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Sketch:

$$\mathrm{CHSH}_{AB^k} > 2 \iff \gamma_k > \frac{2^{k-1} - \cos(\theta)\prod_{j=1}^{k-1}(1+\sqrt{1-\gamma_j^2})}{\sin(\theta)}$$

So for $\epsilon > 0$ set

$$\gamma_k := \begin{cases} (1+\epsilon)\frac{2^{k-1}-\cos(\theta)\prod_{j=1}^{k-1}(1+\sqrt{1-\gamma_j^2})}{\sin(\theta)} & \text{if } 0 \leq \gamma_{k-1} \leq 1 \\ \infty & \text{otherwise} \end{cases}$$

Show you can always choose θ small enough such that $0 < \gamma_1 < \cdots < \gamma_n < 1$.

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- Many works have investigated limitations in the sequential scenario and found strong limitations:
  - Steering: [SHDH$^+$19, SDMM18]
  - Entanglement witnessing: [BMSS18]
  - Other Bell-inequalities: [KP19, DGS$^+$19]
  - Tripartite settings: [SDS$^+$19, MDG$^+$20]

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- Can we translate sequential schemes into a practical advantage?
- Scenario where we also have a sequence of Alices?

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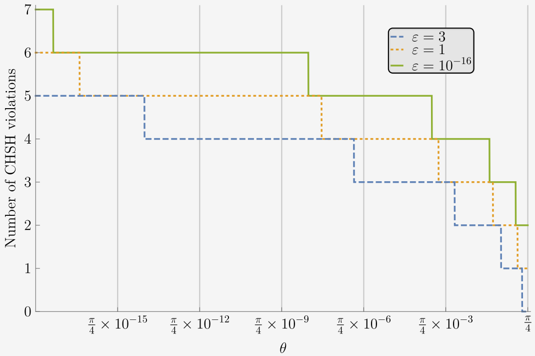
Unbounded violations don't scale so well

We have to pick smaller and smaller θ to allow more Bob's to violate.
Analytical and numerical evidence to suggest θ must decrease double exponentially fast with n.



Pretty bad for the CHSH violations...

$$\mathrm{CHSH}_{AB^n} < 2 + 2^{2-n}\theta.$$