Arbitrarily many independent observers can share the nonlocality of a single maximally entangled qubit pair

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joint work with Roger Colbeck[2]

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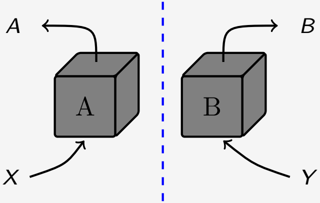
Motivation I

Bell-nonlocality



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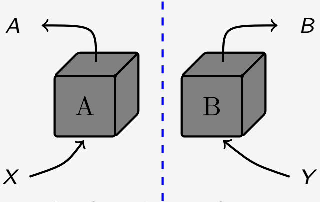

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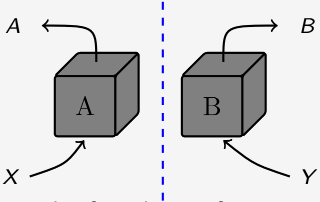

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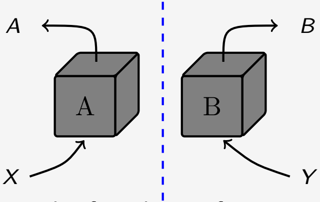
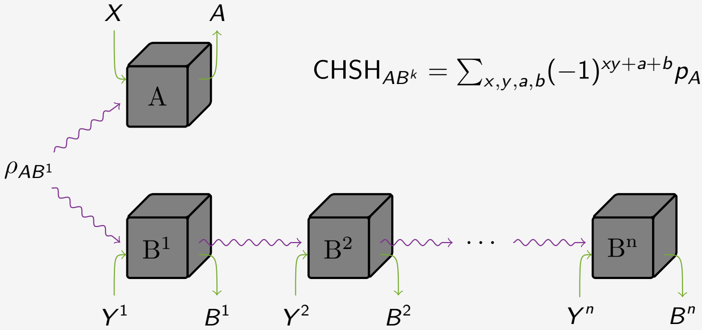


- Nonlocal correlations are the foundation for many device independent protocols
- There are measurements that do not destroy the entanglement between the two halves of the state.
- Can we use this remaining entanglement to generate more nonlocal correlations?

The scenario

We focus on the following scenario introduced in [SGGP15].



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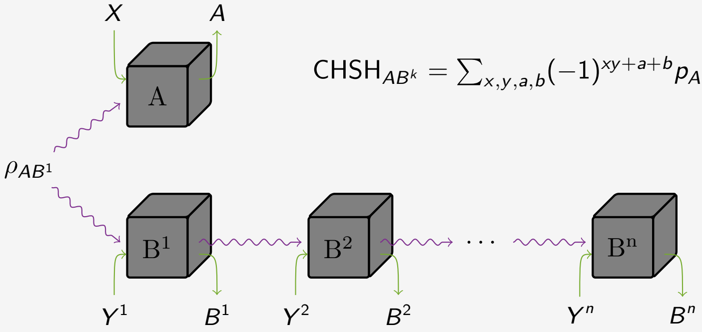

$$\text{CHSH}_{AB^k} = \sum_{x,y,a,b}(-1)^{xy+a+b}p_{AB^k}(a,b|x,y)$$

- All inputs/outputs are binary - inputs chosen uniformly.
- Quantum state passed to next Bob (but not the input/output information)

$$\rho_{AB^n} = \frac{1}{2}\sum_{b_{n-1}y_{n-1}}(I \otimes F_{b_{n-1}|y_{n-1}}^{1/2})\rho_{AB^{n-1}}(I \otimes F_{b_{n-1}|y_{n-1}}^{1/2})$$

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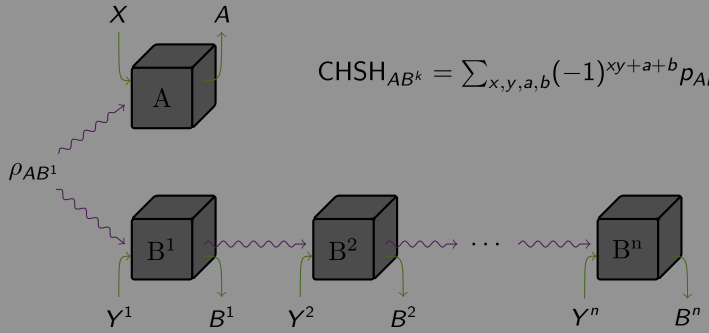

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- All
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Main question

Suppose Alice and Bob[1] share the state ρ_{AB^1}. Then what is the maximum number of Bob's that can achieve an expected CHSH violation with Alice?

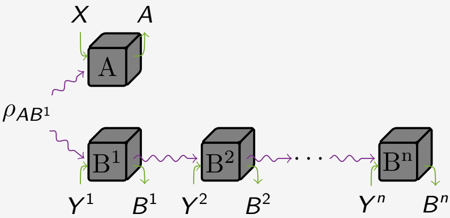
$b_{n-1} y_{n-1}$

The scenario II

Previous works:

Investigated $\rho_{AB^1} = |\Phi^+\rangle\langle\Phi^+|$

- [SGGP15]: heavily biased inputs $\implies$ unbounded #-violations.



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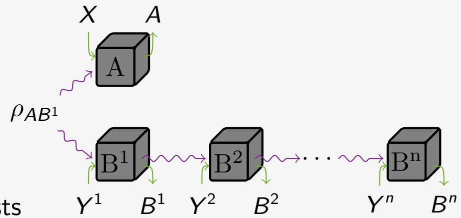

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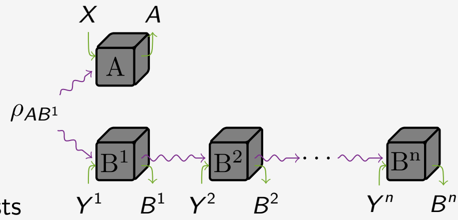

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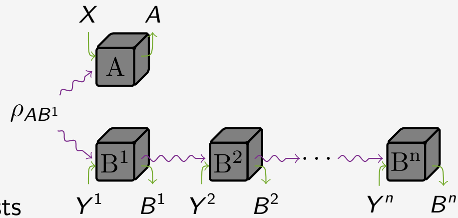

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Here we show the statement is false

- Construct for any $n \in \mathbb{N}$ an explicit n-Bob measurement strategy.
- Extend strategy to a larger class of two-qubit states

The strategy

We consider qubit POVMs $\{M, I - M\}$ with

$$M = I/2 + \gamma(\cos(\varphi)\sigma_z + \sin(\varphi)\sigma_x)/2,$$

where $\varphi \in [-\pi, \pi]$ and $\gamma \in [0, 1]$ is the *sharpness*.

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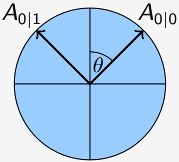
Alice's measurements: For $\theta \in (0, \pi/4]$

$$A_{0|0} = \frac{I + \cos(\theta)\sigma_z + \sin(\theta)\sigma_x}{2}$$

and

$$A_{0|1} = \frac{I + \cos(\theta)\sigma_z - \sin(\theta)\sigma_x}{2}.$$



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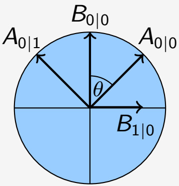

Bob^k's measurements: For $\gamma_k \in (0, 1)$

$$B^k_{0|0} = \frac{I + \sigma_z}{2}$$

and

$$B^k_{0|1} = \frac{I + \gamma_k\sigma_x}{2}.$$

The strategy II

If Alice and Bob[1] start with the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ then

$$\mathrm{CHSH}_{AB^k} = 2^{2-k}\left(\gamma_k \sin(\theta) + \cos(\theta)\prod_{j=1}^{k-1}\left(1+\sqrt{1-\gamma_j^2}\right)\right).$$

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Theorem

For any $n \in \mathbb{N}$ *there exists* $\theta \in (0, \pi/4]$ *and* $(\gamma_1, \ldots, \gamma_n) \in (0,1)^n$ *such that* $\mathrm{CHSH}_{AB^k} > 2$ *for all* $1 \leq k \leq n$.

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Sketch:

$$\mathrm{CHSH}_{AB^k} > 2 \iff \gamma_k > \frac{2^{k-1} - \cos(\theta)\prod_{j=1}^{k-1}(1+\sqrt{1-\gamma_j^2})}{\sin(\theta)}$$

So for $\epsilon > 0$ set

$$\gamma_k := \begin{cases} (1+\epsilon)\frac{2^{k-1} - \cos(\theta)\prod_{j=1}^{k-1}(1+\sqrt{1-\gamma_j^2})}{\sin(\theta)} & \text{if } 0 \leq \gamma_{k-1} \leq 1 \\ \infty & \text{otherwise} \end{cases}$$

Show you can always choose θ small enough such that $0 < \gamma_1 < \cdots < \gamma_n < 1$.

Extension to general two-qubit states

For a general two-qubit state ρ_{AB^1} we give a strategy achieving

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$(T)_{ij} = \mathrm{Tr}\left[\rho(\sigma_i \otimes \sigma_j)\right]$

- No bound on #-violations when $s_1 = 1$!
- Includes all pure entangled two-qubit states.

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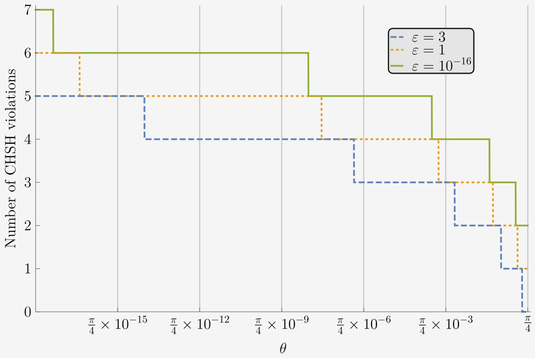

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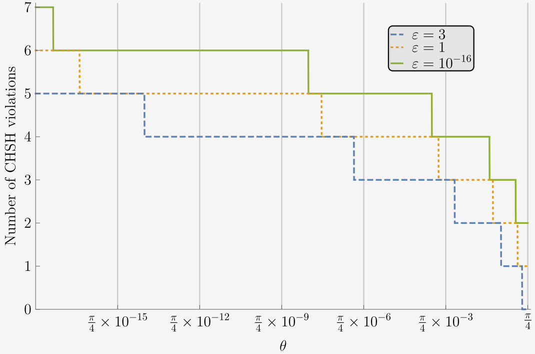

Pretty bad for the CHSH violations. . .

$$\mathrm{CHSH}_{AB^n} < 2 + 2^{2-n}\theta.$$

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    - Entanglement witnessing: [BMSS18]
    - Other Bell-inequalities: [KP19, DGS$^+$19]
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- Scenario where we also have a sequence of Alices?

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