Device-independent lower bounds on the conditional von Neumann entropy

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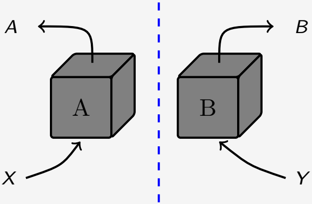
ENS DE LYON

The problem

Observe $P_{AB|XY}$



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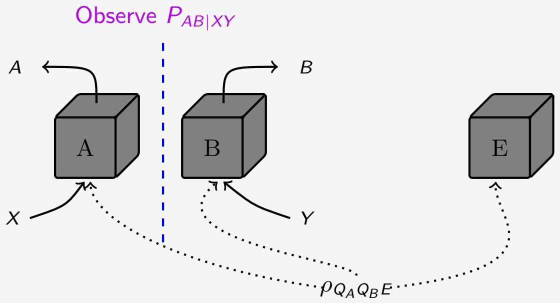

- Foundation for randomness expansion / key-distribution protocols!

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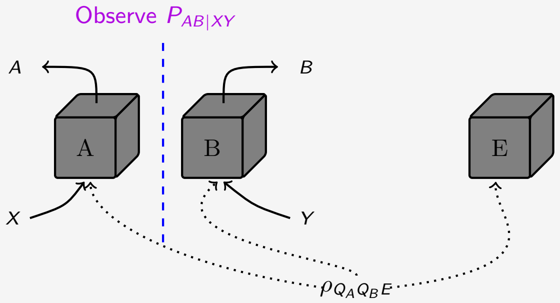

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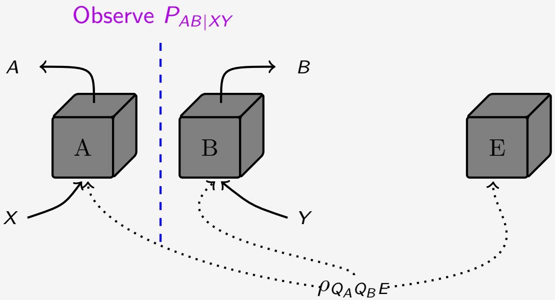

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$$\text{minimize} \quad H(A|E)$$

over **all** devices compatible with statistics.

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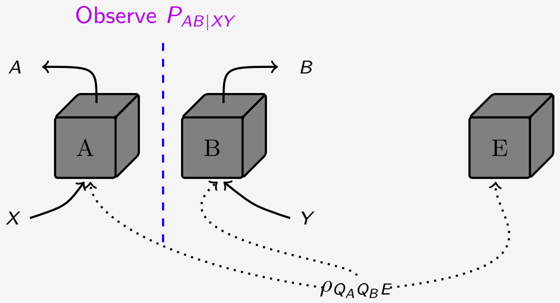

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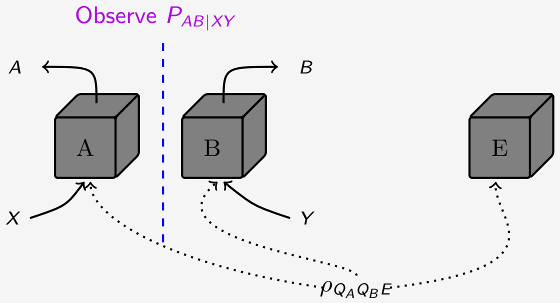

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- Difficult to solve – nonconvex / unbounded dimension

Our approach

- Define a sequence

$$H_m(\rho) = \inf_{Z_1,\ldots,Z_m \in B(H)} \mathrm{Tr}\left[\rho\ q(Z_1,\ldots,Z_m)\right] \tag{1}$$

such that $H_m \leq H$ and $H_m \to H$ as $m \to \infty$.

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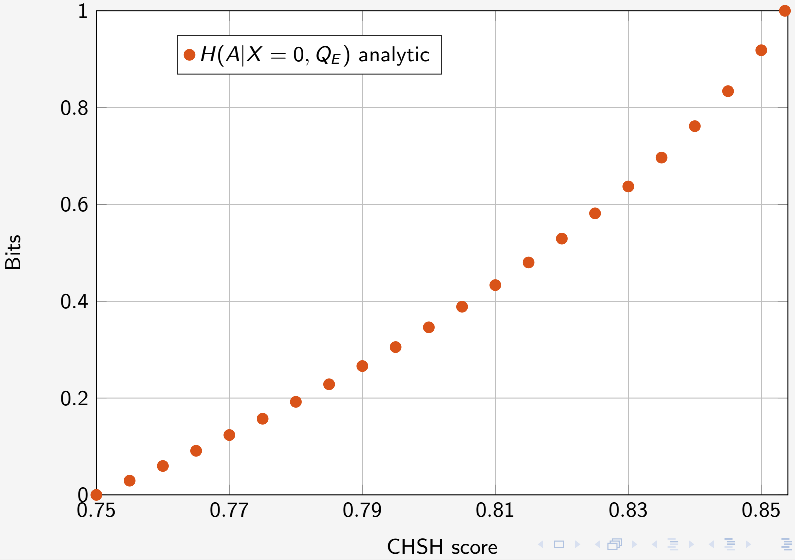
- $\inf H_m$ efficiently approximated by semidefinite programming [NPA].

- close to optimal / more efficient / wider scope

Results I – Recovering tight bounds for the CHSH game

Bounding $\inf H(A|X=0, Q_E)$



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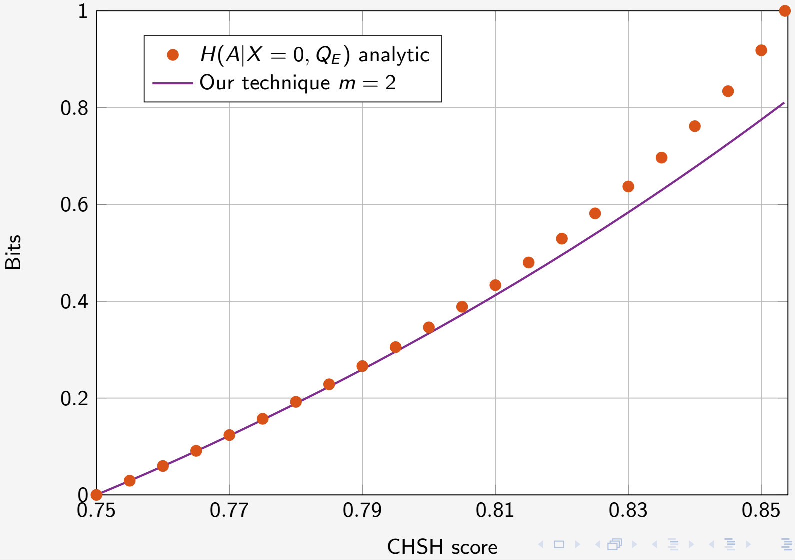

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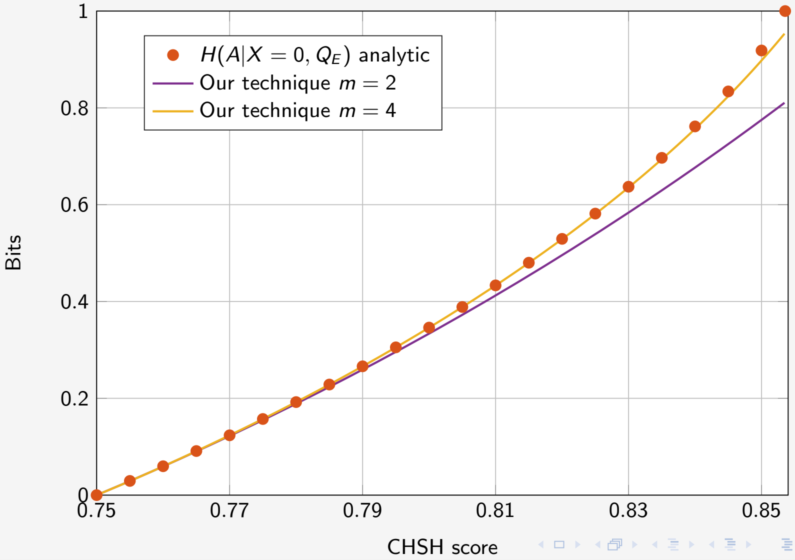

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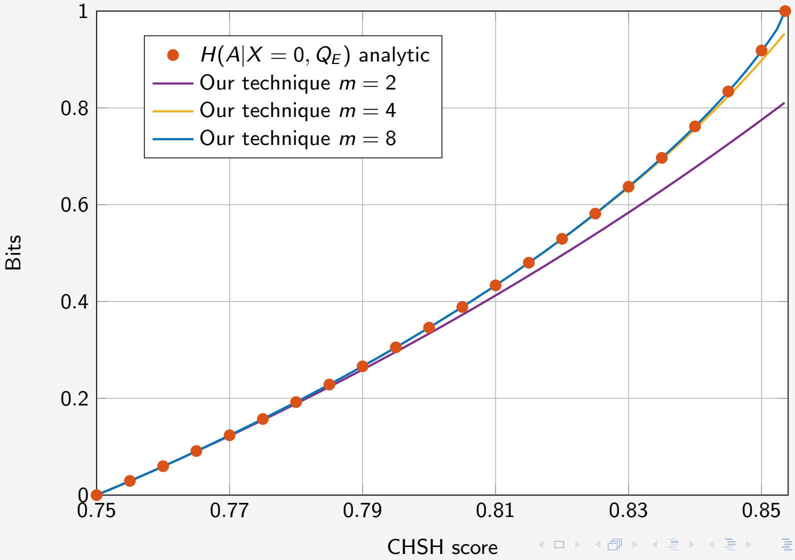

Results III – Improved DIQKD rates

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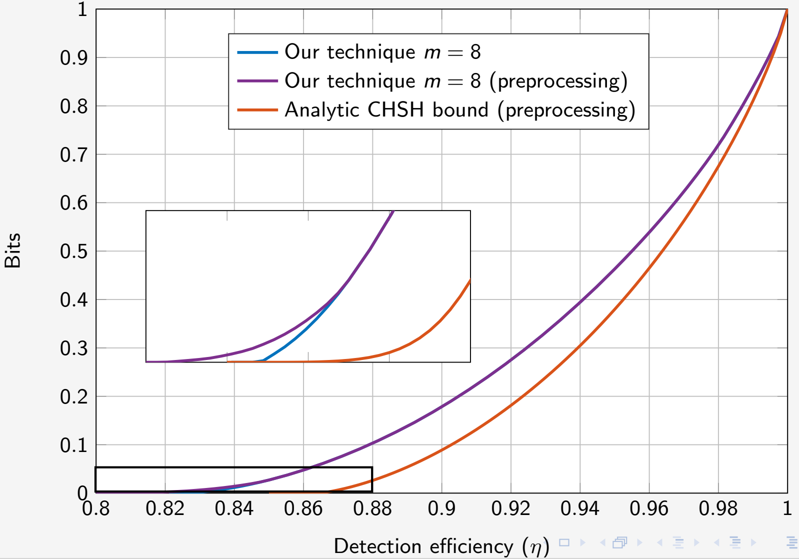

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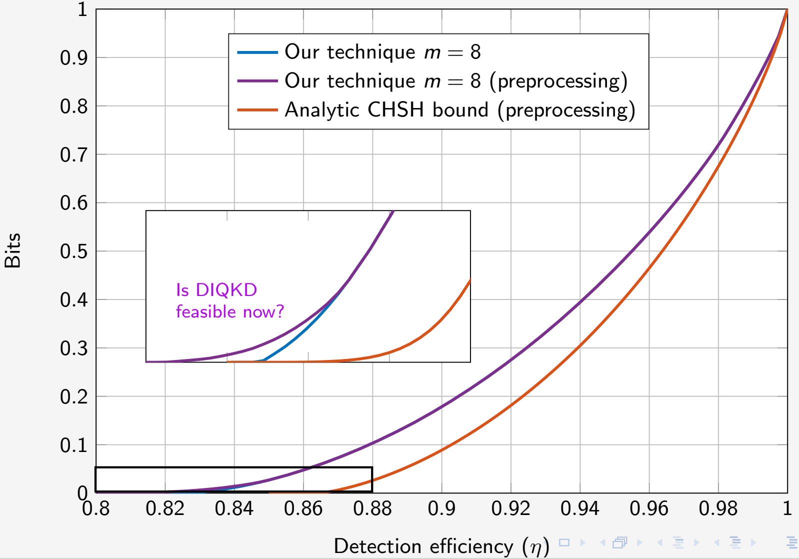

Thanks for listening!